Stochastic Claims Reserving

Methods in Insurance
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Dedicated to
Alessia, Luisa and Anja
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For a very long time estimating claims reserves was understood as a pure algorithmic method to determine the insurance liabilities. It was only about 30 years ago that actuaries have realized that also claims reserving includes stochastic modelling. Due to this reason stochastic claims reserving modelling is still only poorly understood and treated in the education and in the industry. At ETH Zurich we have decided to close this gap and give a whole lecture (one term) on the subject stochastic claims reserving methods in insurance. After this decision we went to the library to look for appropriate lecture notes. Soon it became clear that there is no appropriate text for our lecture and we started the ambitious project of writing our own notes.

The present literature on claims reserving can be divided into several subgroups. The first subgroup consists of various claims reserving manuals for practitioners. These manuals mainly belong to the old school and describe the application of different algorithms, how to clean and smoothen data, how to choose appropriate parameters, etc. Most of these guidelines focus on specific data sets and give proposals for the estimation of the expected outstanding claims liabilities. They are mainly based on practical experience and mathematical aspects and stochastic modelling are only poorly treated in these notes.

Another subgroup consists of books on non-life insurance mathematics. There are several books and lecture notes that treat the basic claims reserving methods in a chapter or a section. Mostly, these chapters give only a very short introduction into the subject without going into depth.

Finally, there is an enormous number of recent scientific papers in various different actuarial journals. Some of these papers are based on sound mathematics and statistical methods, others are less rigorous. Most of them use different notations and it is often difficult to see in which aspects two papers are different and in which they are not. In consolidated book form there is only very few literature that focuses on stochastic aspects.

Based on this background we started our project of writing a book on stochastic claims reserving methods. Our goal was to get a correct and consistent text that unifies different notations and approaches and gives an overview on the contributions that have attracted our attention. We initially started our first lecture on stochastic claims reserving methods with a text that had about 150 pages, but which was still growing dramatically. This has soon led to the decision to limit the scope of this book to probabilistic aspects and calculations and not to include other or more advanced questions like data cleaning, statistical topics, solvency considerations, market-values of reserves etc. which itself would make for at least another two books. So in the present book we define several different stochastic models that are used for claims reserving. Stochastic properties of these models are derived and analysed,
that is, cashflow prediction is studied within these models and measures for analysing the prediction uncertainties are derived.

Initially, we had in mind an audience having a background typical of someone with a degree in mathematics, statistics or actuarial sciences. But to make the book a valuable reference we have decided to also include more advanced topics and calculations. Still we believe that most results should be understood by a modern trained actuary solving (complex) problems for his company. Furthermore, we recognize that we have learned a lot of new methods and techniques during the writing of this book, which was often very enlightening.

We also recognize that the book does not give a complete survey on the existing literature. Moreover, we would also like to mention that it was often very difficult to track the correct chronological and historical roots to all the results developed. We have tried to be as accurate as possible. For all omissions and also in the light of the fast development of research we apologize to all the authors of such missing papers.
Of course the writing of this book was only possible with the support of several individuals. This work would not have been possible without the grant and generosity of our employers, ETH Zurich and University of Tübingen. At ETH Zurich and University of Tübingen we were supported by our departments, our collaborators, our PhD students and, of course, by the students who attended our lectures. At the heart of this encouragement there are the three professors, Paul Embrechts, Hans Bühlmann and Eberhard Schaich, it is only due to them that we have found the fruitful environment (personal, mathematical and methodological) for completing our project. Furthermore, we were assisted by our PhD students Daniel Alai, Matthias Degen, Jochen Heberle, Luis Huergo, Dominik Lambrigger, Natalia Lysenko and Ramona Maier as well as by the diploma students that have completed their thesis during the past years.

However, the beginning of this project was much earlier, before we even knew that we were once going to teach on this subject. Mario started his professional career at Winterthur Insurance (today AXA-Winterthur) where he became responsible for claims reserving for all non-life insurance business lines of Winterthur Insurance Switzerland. The claims reserving group with Alois Gisler, Ursin Mayer, Francesco Pagliari and Markus Steiner introduced him into all the practical secrets of claims reserving, peculiarities of the different lines of business, etc. Michael began his professional career at the actuarial department of the Baloise insurance company in Basel where he gained valuable practical working experience in claims reserving. The non-life group with Marie-Thérèse Kohler, Markus Buchwalder, Hans-Kaspar Britt and Roger Fässler familiarized him with claims reserving and its significance for modern solvency capital requirements, premium calculation, risk management etc. It is only due to our former employers and colleagues that we have learned and realized the importance of claims reserving.

In the process of writing this book we have met many other practitioners and researchers on claims reserving at different universities, conferences, industry, through the actuarial organizations, through the referee processes of actuarial journals, etc. Among all of them, this book has especially profited in some or another form from the knowledge of Gareth Peters, Pavel Shevchenko, Peter England, Thomas Mack, Alain-Sol Sznitman.

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M.V.W. and M.M.