Fundamentals of Digital Image Processing

A Practical Approach with Examples in Matlab

Chris Solomon
School of Physical Sciences, University of Kent, Canterbury, UK

Toby Breckon
School of Engineering, Cranfield University, Bedfordshire, UK

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Preface

Scope of this book

This is an introductory text on the science (and art) of image processing. The book also employs the Matlab programming language and toolboxes to illuminate and consolidate some of the elementary but key concepts in modern image processing and pattern recognition.

The authors are firm believers in the old adage, “Hear and forget. . ., See and remember. . ., Do and know”. For most of us, it is through good examples and gently guided experimentation that we really learn. Accordingly, the book has a large number of carefully chosen examples, graded exercises and computer experiments designed to help the reader get a real grasp of the material. All the program code (.m files) used in the book, corresponding to the examples and exercises, are made available to the reader/course instructor and may be downloaded from the book’s dedicated web site – www.fundipbook.com.

Who is this book for?

For undergraduate and graduate students in the technical disciplines, for technical professionals seeking a direct introduction to the field of image processing and for instructors looking to provide a hands-on, structured course. This book intentionally starts with simple material but we also hope that relative experts will nonetheless find some interesting and useful material in the latter parts.

Aims

What then are the specific aims of this book ? Two of the principal aims are –

- To introduce the reader to some of the key concepts and techniques of modern image processing.

- To provide a framework within which these concepts and techniques can be understood by a series of examples, exercises and computer experiments.
These are, perhaps, aims which one might reasonably expect from any book on a technical subject. However, we have one further aim namely to provide the reader with the fastest, most direct route to acquiring a real hands-on understanding of image processing. We hope this book will give you a real fast-start in the field.

Assumptions

We make no assumptions about the reader’s mathematical background beyond that expected at the undergraduate level in the technical sciences – ie reasonable competence in calculus, matrix algebra and basic statistics.

Why write this book?

There are already a number of excellent and comprehensive texts on image processing and pattern recognition and we refer the interested reader to a number in the appendices of this book. There are also some exhaustive and well-written books on the Matlab language. What the authors felt was lacking was an image processing book which combines a simple exposition of principles with a means to quickly test, verify and experiment with them in an instructive and interactive way.

In our experience, formed over a number of years, Matlab and the associated image processing toolbox are extremely well-suited to help achieve this aim. It is simple but powerful and its key feature in this context is that it enables one to concentrate on the image processing concepts and techniques (i.e. the real business at hand) while keeping concerns about programming syntax and data management to a minimum.

What is Matlab?

Matlab is a programming language with an associated set of specialist software toolboxes. It is an industry standard in scientific computing and used worldwide in the scientific, technical, industrial and educational sectors. Matlab is a commercial product and information on licences and their cost can be obtained direct by enquiry at the web-site www.mathworks.com. Many Universities all over the world provide site licenses for their students.

What knowledge of Matlab is required for this book?

Matlab is very much part of this book and we use it extensively to demonstrate how certain processing tasks and approaches can be quickly implemented and tried out in practice. Throughout the book, we offer comments on the Matlab language and the best way to achieve certain image processing tasks in that language. Thus the learning of concepts in image processing and their implementation within Matlab go hand-in-hand in this text.
Is the book any use then if I don’t know Matlab?

Yes. This is fundamentally a book about image processing which aims to make the subject accessible and practical. It is not a book about the Matlab programming language. Although some prior knowledge of Matlab is an advantage and will make the practical implementation easier, we have endeavoured to maintain a self-contained discussion of the concepts which will stand up apart from the computer-based material.

If you have not encountered Matlab before and you wish to get the maximum from this book, please refer to the Matlab and Image Processing primer on the book website (http://www.fundipbook.com). This aims to give you the essentials on Matlab with a strong emphasis on the basic properties and manipulation of images.

Thus, you do not have to be knowledgeable in Matlab to profit from this book.

Practical issues

To carry out the vast majority of the examples and exercises in the book, the reader will need access to a current licence for Matlab and the Image Processing Toolbox only.

Features of this book and future support

This book is accompanied by a dedicated website (http://www.fundipbook.com). The site is intended to act as a point of contact with the authors, as a repository for the code examples (Matlab .m files) used in the book and to host additional supporting materials for the reader and instructor.

About the authors

Chris Solomon gained a B.Sc in theoretical physics from Durham University and a Ph.D in Medical imaging from the Royal Marsden Hospital, University of London. Since 1994, he has been on the Faculty at the School of Physical Sciences where he is currently a Reader in Forensic Imaging. He has broad research interests focussing on evolutionary and genetic algorithms, image processing and statistical learning methods with a special interest in the human face. Chris is also Technical Director of Visionmetric Ltd, a company he founded in 1999 and which is now the UK’s leading provider of facial composite software and training in facial identification to police forces. He has received a number of UK and European awards for technology innovation and commercialisation of academic research.

Toby Breckon holds a Ph.D in Informatics and B.Sc in Artificial Intelligence and Computer Science from the University of Edinburgh. Since 2006 he has been a lecturer in image processing and computer vision in the School of Engineering at Cranfield University. His key research interests in this domain relate to 3D sensing, real-time vision, sensor fusion, visual surveillance and robotic deployment. He is additionally a visiting member of faculty at Ecole Supérieure des Technologies Industrielles Avancées (France) and has held visiting faculty positions in China and Japan. In 2008 he led the development of