Advanced Dynamic-system Simulation
Model-replication Techniques and Monte Carlo Simulation

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Advanced Dynamic-system Simulation
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Simulation is experimentation with models. This book describes new computer programs for interactive modeling and simulation of dynamic systems, such as aerospace vehicles, control systems, and biological systems. Simulation studies for design or research can involve many hundreds of model changes, so programming must be convenient, and computations must be as fast as possible.

This book is about advanced simulation programming and describes many new techniques. We provide only a brief review of routine simulation programming but demonstrate computer software for remarkably fast and respectably large simulation studies on inexpensive personal computers or workstations. For hands-on experiments, the enclosed CD contains an industrial-strength software package rather than a toy demonstration program.\(^1\)

\(^1\)OPEN DESIRE solves up to 40,000 ordinary differential equations under Linux, and up to 20,000 differential equations under Microsoft Windows\(^\text{TM}\), so that one can implement respectable vectorized Monte Carlo studies. The DESIRE language, widely used since 1985, accepts scalar and vector differential equations and difference equations in a natural mathematical notation, for example,

\[
\frac{dx}{dt} = -x \cdot \cos(w \cdot t) + 2.22 \cdot a \cdot x \\
\text{Vector } y = A \cdot x + B \cdot u
\]

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