Modelling Single-name and Multi-name Credit Derivatives

Dominic O’Kane
Modelling Single-name and Multi-name Credit Derivatives
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To Penny, Rory and Fergal.
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