Volume 2 of the second edition of the fully revised and updated Digital Signal and Image Processing using MATLAB® is essentially a collection of examples and exercises which also presents applications of digital signal- or image processing, and techniques which were not touched upon in the previous volume. It will be of particular benefit to readers who already possess a good knowledge of MATLAB®, a command of the fundamental elements of digital signal processing and who are familiar with both the fundamentals of continuous-spectrum spectral analysis and who have a certain mathematical knowledge concerning Hilbert spaces.

More than 200 programs and functions are provided in the MATLAB® language, with useful comments and guidance, to enable numerical experiments to be carried out, thus allowing readers to develop a deeper understanding of both the theoretical and practical aspects of this subject.

Gérard Blanchet is Professor at Ecole Nationale Supérieure des Télécommunications, Paris, France. In addition to his research, teaching and consulting activities, he is the author of several books on automatic control systems, digital signal processing and computer architecture. He also develops tools and methodologies to improve knowledge acquisition in various fields.

Maurice Charbit is Professor at Ecole Nationale Supérieure des Télécommunications, Paris, France, where he teaches several courses in signal processing and digital communications. His research interests include statistics, speech and image processing.
Digital Signal and Image Processing using MATLAB®
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Foreword

This book represents the continuation to *Digital Signal and Image Processing: Fundamentals*. It is assumed that the reader possesses a good knowledge of the programming language MATLAB® and a command of the fundamental elements of digital signal processing: the usual transforms (the Discrete Time Fourier Transform (DTFT), the Discrete Fourier Transform and the $z$-Transform), the properties of deterministic and random signals, and digital filtering. Readers will also need to be familiar with the fundamentals of continuous-spectrum spectral analysis and have a certain amount of mathematical knowledge concerning vector spaces.

In order to prevent the reading becoming a penance, we will offer a few reminders of the basics wherever necessary. This book is essentially a collection of examples, exercises and case studies. It also presents applications of digital signal- or image processing, and techniques which were not touched upon in the previous volume.

**Recap on digital signal processing**

This section is devoted to the definitions and properties of the fundamental transforms used in digital signal processing: *Fourier transform*, *discrete time Fourier transform* and *discrete Fourier transform*. It concludes with a classic example which enables us to put some known results into practice.

**Filter implementation**

This section deals with the structures of filters, the introduction of parallelism into the filtering operations (block filtering and filter banks) and, by way of an example, the Parks–McClellan method for FIR filter synthesis (*finite impulse response*).

**Image processing**

The section given over to images offers a few geometrical concepts relating to the representation of 3D objects in a 2D space. Therein, we deal with problems
of calibration of cameras. In addition, image compression is also discussed, with the use of examples (pyramidal decompositions, lifting scheme).

**Digital calculus and simulation**

This section deals with the algorithms used in most domains in digital processing, and therefore far beyond mere signal processing. It only touches on the domain using a few examples of methods applied to problems of simulation, resolution of differential equations, zero-seeking, interpolation and iterative methods for solving linear systems.

**Speech processing**

After a brief introduction to speech production, we will discuss the representation of a speech signal by an autoregressive model, and its application to compression. Next we will give the descriptions of the techniques widely used in this field (*Dynamic Time Warping* and *PSOLA*) and, finally, an example of application with “decrackling” for audio recordings.

**Selected topics**

This last chapter presents case studies that go a little further in depth than the examples described in the previous sections. “Tracking the cardiac rhythm of the fetus” and “Extracting the contour of a coin” are classic examples of the application of the least squares method. Principal component analysis and linear discriminant analysis are basic methods for the classification of objects (in a very broad sense).

The section devoted to optimization under constraints could have been part of the section on numerical methods. The method of Lagrange multipliers is encountered in a multitude of applications. In terms of applications, we present the case of optimization of a stock portfolio.

We conclude with the example of the Viterbi algorithm for the hard decoding of convolutional codes. This algorithm is, in fact, a particular case for searching for the shortest possible path in a lattice.
Notations and Abbreviations

∅ empty set
\[ \sum_{k,n} = \sum_k \sum_n \]
\[ \text{rect}_T(t) = \begin{cases} 
1 & \text{when } |t| < T/2 \\
0 & \text{otherwise}
\end{cases} \]
\[ \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \]
\[ 1(x \in A) = \begin{cases} 
1 & \text{when } x \in A \\
0 & \text{otherwise}
\end{cases} \quad \text{(indicator function of } A) \]
\[ (a,b] = \{ x : a < x \leq b \} \]
\[ \delta(t) = \begin{cases} 
\text{Dirac distribution when } t \in \mathbb{R} \\
\text{Kronecker symbol when } t \in \mathbb{Z}
\end{cases} \]
\[ \text{Re}(z) \quad \text{real part of } z \]
\[ \text{Im}(z) \quad \text{imaginary part of } z \]
\[ |x| \quad \text{integer part of } x \]
\[ i \text{ or } j = \sqrt{-1} \]
x(t) \rightleftharpoons X(f) \quad \text{Fourier transform}
\[ (x \ast y)(t) \quad \text{continuous time convolution} \]
\[ = \int_{\mathbb{R}} x(u)y(t-u)du \]
\[ (x \ast y)(t) \quad \text{discrete time convolution} \]
\[ = \sum_{u \in \mathbb{Z}} x(u)y(t-u) = \sum_{u \in \mathbb{Z}} x(t-u)y(u) \]
\[ y^{(n)}(t) = \frac{d^n y(t)}{dt^n}, \text{nth order derivative} \]
<table>
<thead>
<tr>
<th>Symbol/Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{x} ) or ( \bar{\mathbf{x}} )</td>
<td>vector ( \mathbf{x} )</td>
</tr>
<tr>
<td>( \mathbf{I}_N )</td>
<td>((N \times N))-dimension identity matrix</td>
</tr>
<tr>
<td>( \mathbf{A}^* )</td>
<td>complex conjugate of ( \mathbf{A} )</td>
</tr>
<tr>
<td>( \mathbf{A}^T )</td>
<td>transpose of ( \mathbf{A} )</td>
</tr>
<tr>
<td>( \mathbf{A}^H )</td>
<td>transpose-conjugate of ( \mathbf{A} )</td>
</tr>
<tr>
<td>( \mathbf{A}^{-1} )</td>
<td>inverse matrix of ( \mathbf{A} )</td>
</tr>
<tr>
<td>( \mathbf{A}^# )</td>
<td>pseudo-inverse matrix of ( \mathbf{A} )</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>( \mathbb{P} { X \in A } )</td>
<td>probability that ( X \in A )</td>
</tr>
<tr>
<td>( \mathbb{E} { X } )</td>
<td>expectation value of ( X )</td>
</tr>
<tr>
<td>( X_c = X - \mathbb{E} { X } )</td>
<td>zero-mean random variable</td>
</tr>
<tr>
<td>( \text{var} { X } = \mathbb{E} {</td>
<td>X_c</td>
</tr>
<tr>
<td>( \mathbb{E} { X</td>
<td>Y } )</td>
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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>ADPCM</td>
<td>Adaptive Differential PCM</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>ARMA</td>
<td>AR and MA</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>bps</td>
<td>Bits per second</td>
</tr>
<tr>
<td>cdf</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>CF</td>
<td>Clipping Factor</td>
</tr>
<tr>
<td>CZT</td>
<td>Causal ( z )-Transform</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital to Analog Converter</td>
</tr>
<tr>
<td>DCT</td>
<td>Discrete Cosine Transform</td>
</tr>
<tr>
<td>d.e./de</td>
<td>Difference equation</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DTFT</td>
<td>Discrete Time Fourier Transform</td>
</tr>
<tr>
<td>DTMF</td>
<td>Dual Tone Multi-Frequency</td>
</tr>
<tr>
<td>dsp</td>
<td>Digital signal processing/processor</td>
</tr>
<tr>
<td>e.s.d./esd</td>
<td>Energy spectral density</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FT</td>
<td>Continuous Time Fourier Transform</td>
</tr>
</tbody>
</table>
IDFT  Inverse Discrete Fourier Transform
i.i.d./iid  Independent and Identically Distributed
IIR  Infinite Impulse Response
ISI  InterSymbol Interference
LDA  Linear discriminant analysis
lms  Least mean squares
MA  Moving Average
MAC  Multiplication ACcumulation
OTF  Optical Transfer Function
PAM  Pulse Amplitude Modulation
PCA  Principal Component Analysis
p.d.  Probability Distribution
ppi  Points per Inch
p.s.d./PSD  Power Spectral Density
PSF  Point Spread Function
PSK  Phase Shift Keying
QAM  Quadrature Amplitude Modulation
rls  Recursive least squares
rms  Root mean square
r.p./rp  Random process
SNR  Signal to Noise Ratio
r.v./rv  Random variable
STFT  Short Term Fourier Transform
TF  Transfer Function
WSS  Wide (Weak) Sense Stationary (Second Order) Process
ZOH  Zero-Order Hold
ZT  $z$-Transform