Fundamentals of Global Positioning System Receivers
WILEY SERIES IN MICROWAVE AND OPTICAL ENGINEERING

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A complete list of the titles in this series appears at the end of this volume.
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Preface

In this new edition of the book, only minor changes were made to the original nine chapters but three new chapters treat topics of increasing interest to GPS users and equipment developers. One topic, improving the GPS receiver sensitivity may extend their operations into buildings, which is becoming important for emergency rescue and urban warfare. Thus, Chapters 10 and 11 are devoted to the processing of weak signals, as well as the limitations of autonomous GPS receivers. These same approaches are also applicable to GPS receivers in noisy environments and under interference conditions. Other subjects new to this edition, such as using the almanac data to simplify signal acquisition; determining the number of analog-to-digital converter bits required for the GPS receiver to work under strong interference; and, using GPS signals reflected from the ground as an altimeter are covered in Chapter 12.

I constantly discuss technical subjects with Mr. D. Lin and Dr. L. L. Liou, my colleagues at AFRL, and Dr. Y. T. Morton of Miami University. They worked closely with me and made tremendous contributions in this edition. I very much appreciate their help. I would especially like to thank Drs. J. Morton and T. Y. Morton of Miami University and Dr. J. Garrison of Purdue University for reviewing my manuscripts.

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