Elasto-plastic deformation is frequently observed in machines and structures, hence its prediction is an important consideration at the design stage. Elasto-plasticity theories will be increasingly required in the future in response to the development of new and improved industrial technologies. Although various books for elasto-plasticity have been published to date, they focus on infinitesimal elasto-plastic deformation theory. However, modern computational techniques employ an advanced approach to solve problems in this field and much research has taken place in recent years into finite strain elasto-plasticity. This book describes this approach and aims to improve mechanical design techniques in mechanical, civil, structural and aeronautical engineering through the accurate analysis of finite elasto-plastic deformation.

Introduction to Finite Strain Theory for Continuum Elasto-Plasticity presents introductory explanations that can be easily understood by readers with only a basic knowledge of elasto-plasticity, showing physical backgrounds of concepts in detail and derivation processes of almost all equations. The authors address various analytical and numerical finite strain analyses, including new theories developed in recent years, and explain fundamentals including the push-forward and pull-back operations and the Lie derivatives of tensors.

Key features:
• Comprehensively explains finite strain continuum mechanics and explains the finite elasto-plastic constitutive equations
• Discusses numerical issues on stress computation, implementing the numerical algorithms into large-deformation finite element analysis
• Includes numerical examples of boundary-value problems
• Accompanied by a website (www.wiley.com/go/hashiguchi) hosting computer programs for the return-mapping and the consistent tangent moduli of finite elasto-plastic constitutive equations

Introduction to Finite Strain Theory for Continuum Elasto-Plasticity is an ideal reference for research engineers and scientists working with computational solid mechanics and is a suitable graduate text for computational mechanics courses.
INTRODUCTION TO FINITE STRAIN THEORY FOR CONTINUUM ELASTO-PLASTICITY
WILEY SERIES IN COMPUTATIONAL MECHANICS

Series Advisors:
René de Borst
Perumal Nithiarasu
Tayfun E. Tezduyar
Genki Yagawa
Tarek Zohdi

Introduction to Finite Strain Theory for Continuum Elasto-Plasticity
Hashiguchi and Yamakawa
October 2012

Nonlinear Finite Element Analysis of Solids and Structures: Second edition
De Borst, Crisfield, Remmers and Verhoosel
August 2012

An Introduction to Mathematical Modeling: A Course in Mechanics
Oden
November 2011

Computational Mechanics of Discontinua
Munjiza, Knight and Rougier
November 2011

Introduction to Finite Element Analysis: Formulation, Verification and Validation
Szabó and Babuška
March 2011
INTRODUCTION TO FINITE STRAIN THEORY FOR CONTINUUM ELASTO-PLASTICITY

Koichi Hashiguchi
Kyushu University, Japan

Yuki Yamakawa
Tohoku University, Japan
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td></td>
<td>xi</td>
</tr>
<tr>
<td>Series Preface</td>
<td></td>
<td>xv</td>
</tr>
<tr>
<td>Introduction</td>
<td></td>
<td>xvii</td>
</tr>
<tr>
<td>1</td>
<td>Mathematical Preliminaries</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Basic Symbols and Conventions</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Definition of Tensor</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1</td>
<td>Objective Tensor</td>
<td>2</td>
</tr>
<tr>
<td>1.2.2</td>
<td>Quotient Law</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Vector Analysis</td>
<td>5</td>
</tr>
<tr>
<td>1.3.1</td>
<td>Scalar Product</td>
<td>5</td>
</tr>
<tr>
<td>1.3.2</td>
<td>Vector Product</td>
<td>6</td>
</tr>
<tr>
<td>1.3.3</td>
<td>Scalar Triple Product</td>
<td>6</td>
</tr>
<tr>
<td>1.3.4</td>
<td>Vector Triple Product</td>
<td>7</td>
</tr>
<tr>
<td>1.3.5</td>
<td>Reciprocal Vectors</td>
<td>8</td>
</tr>
<tr>
<td>1.3.6</td>
<td>Tensor Product</td>
<td>9</td>
</tr>
<tr>
<td>1.4</td>
<td>Tensor Analysis</td>
<td>9</td>
</tr>
<tr>
<td>1.4.1</td>
<td>Properties of Second-Order Tensor</td>
<td>9</td>
</tr>
<tr>
<td>1.4.2</td>
<td>Tensor Components</td>
<td>10</td>
</tr>
<tr>
<td>1.4.3</td>
<td>Transposed Tensor</td>
<td>11</td>
</tr>
<tr>
<td>1.4.4</td>
<td>Inverse Tensor</td>
<td>12</td>
</tr>
<tr>
<td>1.4.5</td>
<td>Orthogonal Tensor</td>
<td>12</td>
</tr>
<tr>
<td>1.4.6</td>
<td>Tensor Decompositions</td>
<td>15</td>
</tr>
<tr>
<td>1.4.7</td>
<td>Axial Vector</td>
<td>17</td>
</tr>
<tr>
<td>1.4.8</td>
<td>Determinant</td>
<td>20</td>
</tr>
<tr>
<td>1.4.9</td>
<td>On Solutions of Simultaneous Equation</td>
<td>23</td>
</tr>
<tr>
<td>1.4.10</td>
<td>Scalar Triple Products with Invariants</td>
<td>24</td>
</tr>
<tr>
<td>1.4.11</td>
<td>Orthogonal Transformation of Scalar Triple Product</td>
<td>25</td>
</tr>
<tr>
<td>1.4.12</td>
<td>Pseudo Scalar, Vector and Tensor</td>
<td>26</td>
</tr>
<tr>
<td>1.5</td>
<td>Tensor Representations</td>
<td>27</td>
</tr>
<tr>
<td>1.5.1</td>
<td>Tensor Notations</td>
<td>27</td>
</tr>
<tr>
<td>1.5.2</td>
<td>Tensor Components and Transformation Rule</td>
<td>27</td>
</tr>
<tr>
<td>1.5.3</td>
<td>Notations of Tensor Operations</td>
<td>28</td>
</tr>
</tbody>
</table>
1.5.4 Operational Tensors
1.5.5 Isotropic Tensors
1.6 Eigenvalues and Eigenvectors
1.6.1 Eigenvalues and Eigenvectors of Second-Order Tensors
1.6.2 Spectral Representation and Elementary Tensor Functions
1.6.3 Calculation of Eigenvalues and Eigenvectors
1.6.4 Eigenvalues and Vectors of Orthogonal Tensor
1.6.5 Eigenvalues and Vectors of Skew-Symmetric Tensor and Axial Vector
1.6.6 Cayley–Hamilton Theorem
1.7 Polar Decomposition
1.8 Isotropy
1.8.1 Isotropic Material
1.8.2 Representation Theorem of Isotropic Tensor-Valued Tensor Function
1.9 Differential Formulae
1.9.1 Partial Derivatives
1.9.2 Directional Derivatives
1.9.3 Taylor Expansion
1.9.4 Time Derivatives in Lagrangian and Eulerian Descriptions
1.9.5 Derivatives of Tensor Field
1.9.6 Gauss’s Divergence Theorem
1.9.7 Material-Time Derivative of Volume Integration
1.10 Variations and Rates of Geometrical Elements
1.10.1 Variations of Line, Surface and Volume
1.10.2 Rates of Changes of Surface and Volume
1.11 Continuity and Smoothness Conditions
1.11.1 Continuity Condition
1.11.2 Smoothness Condition
1.12 Unconventional Elasto-Plasticity Models

2 General (Curvilinear) Coordinate System
2.1 Primary and Reciprocal Base Vectors
2.2 Metric Tensors
2.3 Representations of Vectors and Tensors
2.4 Physical Components of Vectors and Tensors
2.5 Covariant Derivative of Base Vectors with Christoffel Symbol
2.6 Covariant Derivatives of Scalars, Vectors and Tensors
2.7 Riemann–Christoffel Curvature Tensor
2.8 Relations of Convected and Cartesian Coordinate Descriptions

3 Description of Physical Quantities in Convected Coordinate System
3.1 Necessity for Description in Embedded Coordinate System
3.2 Embedded Base Vectors
3.3 Deformation Gradient Tensor
3.4 Pull-Back and Push-Forward Operations
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.1</td>
<td>Deformation Tensors</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>Strain Tensors</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>4.2.1</td>
<td>Green and Almansi Strain Tensors</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>4.2.2</td>
<td>General Strain Tensors</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>4.2.3</td>
<td>Hencky Strain Tensor</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>4.3</td>
<td>Compatibility Condition</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>Strain Rate and Spin Tensors</td>
<td>146</td>
</tr>
<tr>
<td></td>
<td>4.4.1</td>
<td>Strain Rate and Spin Tensors Based on Velocity Gradient Tensor</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>4.4.2</td>
<td>Strain Rate Tensor Based on General Strain Tensor</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>Representations of Strain Rate and Spin Tensors in Lagrangian and Eulerian Triads</td>
<td>153</td>
</tr>
<tr>
<td></td>
<td>4.6</td>
<td>Decomposition of Deformation Gradient Tensor into Isochoric and Volumetric Parts</td>
<td>158</td>
</tr>
<tr>
<td>5</td>
<td>5.1</td>
<td>Convected Derivative</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td>Corotational Rate</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>5.3</td>
<td>Objectivity</td>
<td>166</td>
</tr>
<tr>
<td>6</td>
<td>6.1</td>
<td>Conservation Laws</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>6.1.1</td>
<td>Basic Conservation Law</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>6.1.2</td>
<td>Conservation Law of Mass</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>6.1.3</td>
<td>Conservation Law of Linear Momentum</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>6.1.4</td>
<td>Conservation Law of Angular Momentum</td>
<td>182</td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td>Stress Tensors</td>
<td>183</td>
</tr>
<tr>
<td></td>
<td>6.2.1</td>
<td>Cauchy Stress Tensor</td>
<td>183</td>
</tr>
<tr>
<td></td>
<td>6.2.2</td>
<td>Symmetry of Cauchy Stress Tensor</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>6.2.3</td>
<td>Various Stress Tensors</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>6.3</td>
<td>Equilibrium Equation</td>
<td>194</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>Equilibrium Equation of Angular Moment</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>6.5</td>
<td>Conservation Law of Energy</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>6.6</td>
<td>Virtual Work Principle</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>6.7</td>
<td>Work Conjugacy</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>6.8</td>
<td>Stress Rate Tensors</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>6.8.1</td>
<td>Contravariant Convected Derivatives</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>6.8.2</td>
<td>Covariant–Contravariant Convected Derivatives</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>6.8.3</td>
<td>Covariant Convected Derivatives</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>6.8.4</td>
<td>Corotational Convected Derivatives</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>6.9</td>
<td>Some Basic Loading Behavior</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td>6.9.1</td>
<td>Uniaxial Loading Followed by Rotation</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td>6.9.2</td>
<td>Simple Shear</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>6.9.3</td>
<td>Combined Loading of Tension and Distortion</td>
<td>220</td>
</tr>
</tbody>
</table>
7 Hyperelasticity

7.1 Hyperelastic Constitutive Equation and Its Rate Form 225
7.2 Examples of Hyperelastic Constitutive Equations 230
  7.2.1 St. Venant–Kirchhoff Elasticity 230
  7.2.2 Modified St. Venant–Kirchhoff Elasticity 231
  7.2.3 Neo-Hookean Elasticity 232
  7.2.4 Modified Neo-Hookean Elasticity (1) 233
  7.2.5 ModifiedNeo-Hookean Elasticity (2) 234
  7.2.6 Modified Neo-Hookean Elasticity (3) 234
  7.2.7 Modified Neo-Hookean Elasticity (4) 234

8 Finite Elasto-Plastic Constitutive Equation 237

8.1 Basic Structures of Finite Elasto-Plasticity 238
8.2 Multiplicative Decomposition 238
8.3 Stress and Deformation Tensors for Multiplicative Decomposition 243
8.4 Incorporation of Nonlinear Kinematic Hardening 244
  8.4.1 Rheological Model for Nonlinear Kinematic Hardening 245
  8.4.2 Multiplicative Decomposition of Plastic Deformation Gradient Tensor 246
8.5 Strain Tensors 249
8.6 Strain Rate and Spin Tensors 252
  8.6.1 Strain Rate and Spin Tensors in Current Configuration 252
  8.6.2 Contravariant–Covariant Pulled-Back Strain Rate and Spin Tensors in Intermediate Configuration 254
  8.6.3 Covariant Pulled-Back Strain Rate and Spin Tensors in Intermediate Configuration 256
  8.6.4 Strain Rate Tensors for Kinematic Hardening 259
8.7 Stress and Kinematic Hardening Variable Tensors 261
8.8 Influences of Superposed Rotations: Objectivity 266
8.9 Hyperelastic Equations for Elastic Deformation and Kinematic Hardening 268
  8.9.1 Hyperelastic Constitutive Equation 268
  8.9.2 Hyperelastic Type Constitutive Equation for Kinematic Hardening 269
8.10 Plastic Constitutive Equations 270
  8.10.1 Normal-Yield and Subloading Surfaces 271
  8.10.2 Consistency Condition 272
  8.10.3 Plastic and Kinematic Hardening Flow Rules 275
  8.10.4 Plastic Strain Rate 277
8.11 Relation between Stress Rate and Strain Rate 278
  8.11.1 Description in Intermediate Configuration 278
  8.11.2 Description in Reference Configuration 278
  8.11.3 Description in Current Configuration 279
8.12 Material Functions of Metals 280
  8.12.1 Strain Energy Function of Elastic Deformation 280
  8.12.2 Strain Energy Function for Kinematic Hardening 281
  8.12.3 Yield Function 282
  8.12.4 Plastic Strain Rate and Kinematic Hardening Strain Rate 283
## 8.13 On the Finite Elasto-Plastic Model in the Current Configuration by the Spectral Representation

284

## 8.14 On the Clausius–Duhem Inequality and the Principle of Maximum Dissipation

285

## Contents

### 9 Computational Methods for Finite Strain Elasto-Plasticity

287

9.1 A Brief Review of Numerical Methods for Finite Strain Elasto-Plasticity

288

9.2 Brief Summary of Model Formulation

289

9.2.1 Constitutive Equations for Elastic Deformation and Isotropic and Kinematic Hardening

289

9.2.2 Normal-Yield and Subloading Functions

291

9.2.3 Plastic Evolution Rules

291

9.2.4 Evolution Rule of Normal-Yield Ratio for Subloading Surface

293

9.3 Transformation to Description in Reference Configuration

293

9.3.1 Constitutive Equations for Elastic Deformation and Isotropic and Kinematic Hardening

293

9.3.2 Normal-Yield and Subloading Functions

294

9.3.3 Plastic Evolution Rules

295

9.3.4 Evolution Rule of Normal-Yield Ratio for Subloading Surface

296

9.4 Time-Integration of Plastic Evolution Rules

296

9.5 Update of Deformation Gradient Tensor

300

9.6 Elastic Predictor Step and Loading Criterion

301

9.7 Plastic Corrector Step by Return-Mapping

304

9.8 Derivation of Jacobian Matrix for Return-Mapping

308

9.8.1 Components of Jacobian Matrix

308

9.8.2 Derivatives of Tensor Exponentials

310

9.8.3 Derivatives of Stresses

312

9.9 Consistent (Algorithmic) Tangent Modulus Tensor

312

9.9.1 Analytical Derivation of Consistent Tangent Modulus Tensor

313

9.9.2 Numerical Computation of Consistent Tangent Modulus Tensor

315

9.10 Numerical Examples

316

9.10.1 Example 1: Strain-Controlled Cyclic Simple Shear Analysis

318

9.10.2 Example 2: Elastic–Plastic Transition

318

9.10.3 Example 3: Large Monotonic Simple Shear Analysis with Kinematic Hardening Model

320

9.10.4 Example 4: Accuracy and Convergence Assessment of Stress-Update Algorithm

322

9.10.5 Example 5: Finite Element Simulation of Large Deflection of Cantilever

326

9.10.6 Example 6: Finite Element Simulation of Combined Tensile, Compressive, and Shear Deformation for Cubic Specimen

330

### 10 Computer Programs

337

10.1 User Instructions and Input File Description

337

10.2 Output File Description

340
10.3 Computer Programs

10.3.1 Structure of Fortran Program returnmap

10.3.2 Main Routine of Program returnmap

10.3.3 Subroutine to Define Common Variables: comvar

10.3.4 Subroutine for Return-Mapping: retmap

10.3.5 Subroutine for Isotropic Hardening Rule: phiso

10.3.6 Subroutine for Numerical Computation of Consistent Tangent Modulus Tensor: tgunum0

A Projection of Area

B Geometrical Interpretation of Strain Rate and Spin Tensors

C Proof for Derivative of Second Invariant of Logarithmic-Deviatoric Deformation Tensor

D Numerical Computation of Tensor Exponential Function and Its Derivative

D.1 Numerical Computation of Tensor Exponential Function

D.2 Fortran Subroutine for Tensor Exponential Function: matexp

D.3 Numerical Computation of Derivative of Tensor Exponential Function

D.4 Fortran Subroutine for Derivative of Tensor Exponential Function: matdex

References

Index
Preface

The first author of this book recently published the book *Elastoplasticity Theory* (2009) which addresses the fundamentals of elasto-plasticity and various plasticity models. It is mainly concerned with the elasto-plastic deformation theory within the framework of the hypoelastic-based plastic constitutive equation. It has been widely adopted and has contributed to the prediction of the elasto-plastic deformation behavior of engineering materials and structures composed of solids such as metals, geomaterials and concretes. However, the hypoelastic-based plastic constitutive equation is premised on the additive decomposition of the strain rate (symmetric part of velocity gradient) into the elastic and the plastic strain rates and the linear relation between the elastic strain rate and stress rate. There is no one-to-one correspondence between the time-integrations of the elastic strain rate and stress rate and the energy may be produced or dissipated during a loading cycle in hypoelastic equation. Therefore, the elastic strain rate does not possess the elastic property in the strict sense so that an error may be induced in large-deformation analysis and accumulated in the cyclic loading analysis. An exact formulation without these defects in the infinitesimal elasto-plasticity theory has been sought in order to respond to recent developments in engineering in the relevant fields, such as mechanical, aeronautic, civil, and architectural engineering.

There has been a great deal of work during the last half century on the finite strain elasto-plasticity theory enabling exact deformation analysis up to large deformation, as represented by the epoch-making works of Oldroyd (1950), Kroner (1959), Lee (1969), Kratochvil (1971), Mandel (1972b), Hill (1978), Dafalias (1985), and Simo (1998). In this body of work the multiplicative decomposition which is the decomposition of the deformation gradient into the elastic and plastic parts, introducing the intermediate configuration obtained by unloading to the stress free-state, was proposed and the elastic part is formulated as a hyperelastic relation based on the elastic strain energy function. Further, the Mandel stress, the work-conjugate plastic velocity gradient with the Mandel stress, the plastic spin, and various physical quantities defined in the intermediate configuration have been introduced. The physical and mathematical foundations for the exact finite elasto-plasticity theory were established by 2000 and the constitutive equation based on these foundations – the hyperelastic-based plastic constitutive equation – has been formulated after 2005. However, a textbook on the hyperelastic-based finite elasto-plasticity theory has not been published to date.

Against this backdrop, the aim of this book is to give a comprehensive explanation of the finite elasto-plasticity theory. First, the classification of elastoplasticity theories from the viewpoint of the relevant range of deformation will be given and the prominence of the hyperelastic-based finite elastoplasticity theory will be explained in Introduction. Exact knowledge of the
basic mechanical ingredients – finite strain (rate) tensors, the Lagrangian and Eulerian tensors, the objectivity of the tensor and the systematic definitions of pull-back and push-forward operations, the Lie derivative and the corotational rate – is required in the formulation of finite strain theory. To this end, descriptions of the physical quantities and relations in the embedded (convected) coordinate system, which turns into the curvilinear coordinate system under the deformation of material, are required, since their physical meanings can be captured clearly by observing them in a coordinate system which not only moves but also deforms and rotates with material itself. In other words, the essentials of continuum mechanics cannot be captured without the incorporation of the general curvilinear coordinate system, although numerous books with ‘continuum mechanics’ in their title and confined to the rectangular coordinate system have been published to date. On the other hand, knowledge of the curvilinear coordinate system is not required for users of finite strain theory. It is sufficient for finite element analyzers using finite strain theory to master the descriptions in the rectangular coordinate system and the relations between Lagrangian and Eulerian tensors through the deformation gradient, for instance. This book, which aims to impart the exact finite strain theory, gives a comprehensive explanation of the mathematical and physical fundamentals required for continuum solid mechanics, and provides a description in the general coordinate system before moving on to an explanation of finite strain theory.

In addition to the above-mentioned issues on the formulation of the constitutive equation, the formulation and implementation of a numerical algorithm for the state-updating calculation are of utmost importance. The state-updating procedure in the computational analysis of elasto-plasticity problems usually requires a proper algorithm for numerical integration of the rate forms of the constitutive laws and the evolution equations. The return-mapping scheme has been developed to a degree of common acceptance in the field of computational plasticity as an effective state-updating procedure for elasto-plastic models. In the numerical analysis of boundary-value problems, a consistent linearization of the weak form of the equilibrium equation and use of the so-called consistent (algorithmic) elasto-plastic tangent modulus tensor are necessary to ensure effectiveness and robustness of the iterative solution procedure. A Fortran program for the return-mapping and the consistent tangent modulus tensor which can readily be implemented in finite element codes is provided along with detailed explanation and user instructions so that readers will be able to carry out deformation analysis using the finite elasto-plasticity theory by themselves.

Chapters 1–7 were written by the first author, Chapter 8 by both authors, and Chapters 9 and 10 by the second author in close collaboration, and the computer programming and calculations were performed by the second author based on the theory formulated by both authors in Chapter 8. The authors hope that readers of this book will capture the fundamentals of the finite elasto-plasticity theory and will contribute to the development of mechanical designs of machinery and structures in the field of engineering practice by applying the theories addressed in this book. A reader is apt to give up reading a book if he encounters matter which is difficult to understand. For this reason, explanations of physical concepts in elasto-plasticity are given, and formulations and derivations/transformations for all equations are given without abbreviation for Chapters 1–8 for the basic formulations of finite strain theory. This is not a complete book on the finite elasto-plasticity theory, but the authors will be quite satisfied if it provides a foundation for further development of the theory by stimulating the curiosity of young researchers and it is applied widely to the analyses of engineering
problems in practice. In addition, the authors hope that it will be followed by a variety of books on the finite elasto-plasticity.

The first author is deeply indebted to Professor B. Raniecki of the Institute of Fundamental Technological Research, Poland, for valuable suggestions and comments on solid mechanics, who has visited several times Kyushu University. His lecture notes on solid mechanics and regular private communications and advice have been valuable in writing some parts of this book. He wishes also to express his gratitude to Professor O.T. Bruhns of the Ruhr University, Bochum, Germany, and Professor H. Petryk of the Institute of Fundamental Technological Research, Poland, for valuable comments and for notes on their lectures on continuum mechanics delivered at Kyushu University. The second author would like to express sincere gratitude to Professor Kiyohiro Ikeda of Tohoku University for valuable suggestions and comments on nonlinear mechanics. He is also grateful to Professor Kenjiro Terada of Tohoku University for providing enlightening advices on numerical methods for finite strain elasto-plasticity. He also thanks Dr. Ikumu Watanabe of National Institute for Materials Science, Japan, for helpful advices on numerical methods for finite strain elasto-plasticity. The enthusiastic support of Dr. Keisuke Sato of Terrabyte Inc., Japan, and Shoya Nakaichi, Toshimitsu Fujisawa, Yosuke Yamaguchi and Yutaka Chida at Tohoku University in development and implementation of the numerical code is most appreciated. The authors thank Professor S. Reese of RWTH Aachen University, Professor J. Ihlemann and Professor A.V. Shutov of Chemnitz University, Germany, and Professor M. Wallin of Lund University, Sweden for valuable suggestions and for imparting relevant articles on finite strain theory to the authors.

Koichi Hashiguchi
Yuki Yamakawa
February 2012