Understanding the Mathematics of Personal Finance
To all the people struggling to understand the calculations behind the various financial instruments they encounter: I hope this book helps.
Contents

Preface xi
Acknowledgments xv
List of Abbreviations xvii

1. Background Mathematics 1
   1.1 Arithmetic, Notation, and Formulas 1
   1.2 Minus (Negative) Signs 5
   1.3 Lists and Subscripted Variables 6
   1.4 Changes 9
   1.5 Exponents 11
   1.6 Summations 12
   1.7 Graphs and Charts 13
   1.8 Approximations 18
   1.9 Rates—Average and Instantaneous 19
   1.10 Inequalities and Ranges of Numbers 22
   Problems 23

2. Compound Interest 26
   2.1 Some Mathematics 30
   2.2 My Website Spreadsheet 31
   2.3 Online Calculators 32
   2.4 Scaling 32
   2.5 Proration—Working Inside a Compounding Interval 33
   2.6 Initial Charges and Effective Interest Rate 34
   2.7 In the Limit—Continuous Compounding 35
   Problems 37

3. Loan Amortization and Savings 38
   3.1 Loans 38
   3.2 Calculating the Payment Amount 44
   3.3 Paying off a Loan Very Slowly 45
   3.4 My Website Spreadsheet 46
   3.5 A Note About Total Interest for a Year 47
   3.6 Online Calculators 48
   3.7 Loans with First Payment Due Immediately 48
   3.8 Irregular Payments 49
Contents

3.9 Regular Savings 50
Problems 52

4. Mortgages 54

4.1 Online Calculators 55
4.2 Fixed Rate Mortgages 55
4.3 Adjustable Rate Mortgages (ARMs) 57
4.4 Balloon Loans 59
4.5 Up-Front Costs 59
Problems 60

5. Prepayment Penalties 61

5.1 Rule of 78 63
5.2 Other Prepayment Penalties 66
Problems 68

6. Credit Cards 69

6.1 Credit Card Statements 70
6.2 Transfers 73
6.3 Payment Allocation 75
6.4 Daily Balance 76
6.5 Some Calculation Examples 76
6.6 Grace Period 81
6.7 Changing Interest Rates 81
6.8 A Bankruptcy Spiral 81
6.9 Minimum Payment 83
6.10 Other Interest Calculation Approaches 83
6.11 Debit Cards—Something Completely Different 83
Problems 84

7. Present Value 86

7.1 Online Calculators 88
7.2 Doing It with My Spreadsheet 88
7.3 The Effect of Interest Rates on Present Value Calculations 90
7.4 Why This All Matters 91
7.5 A Very Involved Example: Writing Your Own Spreadsheet 91
7.6 Future Value 93
7.7 Present Value of Prepayment Penalties 94
Problems 95

8. Comparing Loans 96

8.1 Up-Front Costs 99
8.2 Adjustable Rate Mortgages (ARMs)  100
8.3 A Few Last Words  102
Problems  103

9. Taxation and Inflation  105
9.1 Understanding Personal Federal Income Tax Rates  105
9.2 Online Tax Calculators  111
9.3 Taxation of Earned Interest  111
9.4 Deductible Interest  112
9.5 Deferred Taxation Savings  112
9.6 Online Deferred Taxation Plan Calculators  115
9.7 Inflation  115
Problems  118

10. Life Insurance  119
10.1 What Is an Insurance Policy?  119
10.2 Probability  120
10.3 Introduction to the Life Tables  126
10.4 Expected Values  128
10.5 Term Insurance  130
10.6 Time Payments  133
10.7 Decreasing Term Insurance  134
10.8 Insurance for the Rest of Your Life  134
10.9 Whole Life Insurance  136
10.10 Breaking Down the Year  139
Problems  142

11. Annuities  144
11.1 A Benchmark Savings Plan  145
11.2 Immediate Annuity with Period Certain  145
11.3 Deferred Annuities  148
11.4 Life Annuities  150
11.5 Payments for Couples  153
11.6 Online Calculators  156
11.7 Variable Annuities  156
Problems  157

12. Reverse Mortgages and Viatical Settlements  158
12.1 Reverse Mortgages  158
12.2 Viatical Settlements  162
Problems  163
13. **Investing: Risk versus Reward**

- 13.1 Stocks 165
- 13.2 Portfolios 167
- 13.3 Calculators 171
- 13.4 Dollar Cost Averaging 173
- 13.5 Short Sales 174
- 13.6 Stock Dividends 174
- 13.7 Bonds 175
- 13.8 Options 175
- 13.9 Online Calculators and Listings 179
- 13.10 Ponzi Schemes and Other Scams 180

Problems 185
Reference 186

14 **Gambling**

- 14.1 Probability and Odds 187
- 14.2 Probability and Expected Return 188
- 14.3 Pari-Mutuel Betting 193

Problems 197

15 **Spreadsheet Calculators**

- 15.1 Introduction to the Spreadsheets 198
- 15.2 Some Programming Notes 205

16 **Solutions**

- 16.1 Chapter 1 207
- 16.2 Chapter 2 210
- 16.3 Chapter 3 213
- 16.4 Chapter 4 215
- 16.5 Chapter 5 216
- 16.6 Chapter 6 218
- 16.7 Chapter 7 220
- 16.8 Chapter 8 221
- 16.9 Chapter 9 223
- 16.10 Chapter 10 226
- 16.11 Chapter 11 229
- 16.12 Chapter 12 233
- 16.13 Chapter 13 234
- 16.14 Chapter 14 237

Index 240
What is personal finance? An informal definition is “how you interact with money.” Among the subcategories of personal finance are topics such as budgeting, saving, borrowing, investing, gambling, and buying and selling real estate. Many books, courses, professional advisors, and software programs are available to help you optimize your path through your financial life.

This book is about various forms of borrowing and saving money, and includes some discussion of investing money. Borrowing money takes many forms, including home mortgage loans, auto loans, and credit card debt. Saving money includes putting money under your mattress, depositing it into a savings bank, and buying certificates of deposit (CDs). Insurance policies can be thought of as a special kind of pooled savings plan whereby many people put money into the same savings account, and this money becomes available to these people when a specified special need (illness, repairing a car, death benefit) unexpectedly arises. Investing is an opportunity to earn more money with your money than a savings bank will give you, but with less certainty about the earnings and, for that matter, less certainty about maintaining your original money than a government-insured savings account would give you.

When you borrow money or, equivalently, take a loan from a person, a bank, a mortgage company, or elsewhere, you will be expected to pay a fee for the use of this money. The amount you borrow is called the principal of your loan and the fee you pay for borrowing the money is called the interest. The amount of interest you have to pay is based upon the principal, the amount of time you have the money, and the prevailing financial conditions. The longer you have this money, the more interest you can expect to pay. In common situations such as a home mortgage or a car loan, you usually repay the loan gradually over a period of time. In this case, calculating the interest gets a little messy because the amount you owe at any given time (the balance) is being reduced due to your payments, while it is simultaneously being increased by the accrual of interest based on your balance at that time. In a properly structured loan, your payments are large enough that the balance decreases after each payment and eventually goes to 0, so that your loan is paid off.

The concepts and calculations for a simple one repayment loan and for multiple payment loans such as mortgages and car loans are the same; it’s just that in the latter cases you have to repeat the same calculations many times. Before the era of spreadsheets on personal computers and the Internet, the complexity of the multiple calculations was so significant that only banks and mortgage companies and other large financial institutions could undertake them. When you took a loan, you would be provided with a table of payment due dates and loan balances (an amortization
table) for your loan. Comparing different loan opportunities was very difficult unless you wanted to spend a lot of time in the library working with books of loan tables.

Today, everybody can easily calculate loan details themselves. Pocket calculators with all the necessary financial functions built-in are inexpensive and easy to use. Users of spreadsheet programs on personal computers can generate their own amortization tables based on the financial functions built into these spreadsheets and/or can build up these formulas from basic principles. Most common financial calculations are available on the Internet (“online”) in the form of simple calculators designed specifically for a single type of problem.

My goal in writing the book is to explain how even the most involved loan scenarios can be understood just by repeated application of the fundamental concept of compound interest, which is the subject of Chapter 2. I’ll show how to calculate everything involved with these loans using a computer spreadsheet program, and whenever possible, I’ll reference some online calculators—particularly those on my own website.

I should mention here that I’m using “loan” as a generic term for one party letting another party use his or her money for some time and expecting interest as compensation. When you take a mortgage loan on a home, you are borrowing the money from somebody. When you put money into a savings bank or purchase a CD (“invest” your money), the bank is borrowing money from you. In terms of the mathematics involved, these are identical situations—you just have to keep track of which way dollars are flowing.

If you loan me money, then I am borrowing money from you and vice versa. In terms of usage, I often see that the terms loan and borrow are used interchangeably. In many situations that you encounter, you’ll simply have to pull the correct meaning out of context. This is unfortunate because each term has a specific meaning; they’re not interchangeable. I will admit that the correct usage can sometimes be confusing—when I take a loan, I’m borrowing money. The person or company that loaned me the money is the lender, and once I’ve borrowed the money I am the debtor.

This is not a book that gives investment or borrowing strategies. I won’t offer suggestions on how to plan for retirement, whether or not you want a reverse mortgage, how to allocate your savings, and so on. My goal is to provide the tools for you to be able to calculate the real costs and/or profits involved in using these various financial instruments and therefore to put you in a position to see for yourself what the best deals are and/or how you could sometimes get yourself into a financial mess.

The most important concept to hold in your mind is that because of interest accruing on borrowed money, the amount of money you owe (or are owed) has a time value to it. One thousand dollars to be paid to you today is worth more than $1,000 to be paid to you a year from now. One thousand dollars to be paid to you a year from now is worth more than $2,000 to be paid to you 20 years from now. You must learn to work with concepts such as future value, which is the amount that some number of dollars today will be worth on a specific date in the future, and also the present value, which is the amount that some number of dollars on some specific date in the future is worth today.
In this book, you will find descriptions of various financial instruments (mortgages, credit card purchases, cash advances, etc.) You will also see how these financial instruments work and how to use the proper analysis tools (primarily the computer spreadsheet) well enough that you can tackle a new situation and come up with the right answers.

There are many computer spreadsheet programs available. Fortunately, they are all very similar in structure, and the instructions I give for my spreadsheets will work on all popular spreadsheet programs.

The spreadsheet calculators used in this book are all available on my website (www.lawrencedworsky.com). Chapter 15 shows you how to get a free spreadsheet program if you need one, how to get to the spreadsheets I’m providing, and a general introduction on how I’m setting them up and how to use and maintain them.

In a sense, this book will never be finished. My website will always be changing. I will improve the existing spreadsheets, adding examples and explanations as well as new capabilities. I will have an up-to-date errata section (that hopefully will be very, very, short). Also, my website has the typical Contact Me capability. This is how I will learn what I haven’t explained well, what relevant facts or scenarios I have overlooked, and so on. I will address all of these matters and put my work on the website as quickly as possible. Interesting problems may become additional problems for the book, posted on the website.

Chapter 1 contains a review of the basic mathematics necessary to understand the book. Most readers shouldn’t find this math difficult. The only new information presented is that the notation isn’t usually what was taught to you in high school. I’ll go through this slowly and carefully. There are powerful notations to properly express calculations that you probably already know how to do. These notations are important because they can describe involved calculations clearly and concisely.

Included in the book are a few sections of mathematical nature that delve a bit more deeply into a topic than does most of the book. These sections are not necessary for a good understanding of the book or use of the calculator spreadsheets and can be skipped if you wish. I’ll clearly state at the beginning of each of these sections that you can skip the section if you don’t want to wrestle with the mathematics.

Lawrence N. Dworsky