VIBRATIONS AND WAVES

George C. King

School of Physics & Astronomy,
The University of Manchester, Manchester, UK
Vibrations and Waves
This book is dedicated to Franz Mandl. I first encountered him as an inspirational teacher when I was an undergraduate. Later, we became colleagues and firm friends at Manchester. Franz was the editor throughout the writing of the book and made many valuable suggestions and comments based upon his wide-ranging knowledge and profound understanding of physics. Discussions with him about the various topics presented in the book were always illuminating and this interaction was one of the joys of writing the book.
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Editors’ Preface to the Manchester Physics Series

The Manchester Physics Series is a series of textbooks at first degree level. It grew out of our experience at the University of Manchester, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course; and that this material is only rarely so arranged as to allow the definition of a short self-contained course. In planning these books we have had two objectives. One was to produce short books so that lecturers would find them attractive for undergraduate courses, and so that students would not be frightened off by their encyclopaedic size or price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second objective was to produce books which allow courses of different lengths and difficulty to be selected with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of latter parts of each volume.

Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each Author’s Preface gives details about the level, prerequisites, etc., of that volume.

The Manchester Physics Series has been very successful since its inception 40 years ago, with total sales of more than a quarter of a million copies. We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, for helpful criticisms and stimulating comments. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, the suggestions of the editors.

Finally we would like to thank our publishers, John Wiley & Sons, Ltd, for their enthusiastic and continued commitment to the Manchester Physics Series.

F. K. Loebinger
F. Mandl
D. J. Sandiford
August 2008
Author’s Preface

Vibrations and waves lie at the heart of many branches of the physical sciences and engineering. Consequently, their study is an essential part of the education of students in these disciplines. This book is based upon an introductory 24-lecture course on vibrations and waves given by the author at the University of Manchester. The course was attended by first-year undergraduate students taking physics or a joint honours degree course with physics. This book covers the topics given in the course although, in general, it amplifies to some extent the material delivered in the lectures.

The organisation of the book serves to provide a logical progression from the simple harmonic oscillator to waves in continuous media. The first three chapters deal with simple harmonic oscillations in various circumstances while the last four chapters deal with waves in their various forms. The connecting chapter (Chapter 4) deals with coupled oscillators which provide the bridge between waves and the simple harmonic oscillator. Chapter 1 describes simple harmonic motion in some detail. Here the universal importance of the simple harmonic oscillator is emphasised and it is shown how the elegant mathematical description of simple harmonic motion can be applied to a wide range of physical systems. Chapter 2 extends the study of simple harmonic motion to the case where damping forces are present as they invariably are in real physical situations. It also introduces the quality factor $Q$ of an oscillating system. Chapter 3 describes forced oscillations, including the phenomenon of resonance where small forces can produce large oscillations and possibly catastrophic effects when a system is driven at its resonance frequency. Chapter 4 describes coupled oscillations and their representation in terms of the normal modes of the system. As noted above, coupled oscillators pave the way to the understanding of waves in continuous media. Chapter 5 deals with the physical