Fundamentals of Fluid Mechanics and Transport Phenomena

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Preface

The study of fluid mechanics and transfer phenomena in flows involves the association of difficulties which are encountered in different disciplines: thermodynamics, mechanics, thermal conduction, diffusion, chemical reactions, etc. This book is not intended to be an encyclopaedia, and we will thus not endeavour to cover all of the aforementioned disciplines in a detailed fashion. The main objective of the text is to present the study of the movement of fluids and the main consequences in terms of the transfer of mass and heat. The book is the result of many years of teaching and research, both theoretical and applied, in scientific domains which are often considered separately. In effect, the development of new disciplines which are at the same time specialized and universal was very much a characteristic of science in the 20th century. Thus, signal processing, system analysis, numerical analysis, etc. are all autonomous disciplines and indispensable means for students, engineers or researchers working in the domain of fluid mechanics and energetics. In the same way, various domains such as the design of chemical reactors, the study of the stars and meteorology require a solid knowledge of fluid mechanics in addition to that of their specific topics.

This book is primarily aimed at students, engineers and researchers in fluid mechanics and energetics. However, we feel that it can be useful for people working in other disciplines, even if the reading of some of the more theoretical and specialized chapters may be dispensable in this case. The science and technology of the first half of the 20th century was heavily rooted in classical mechanics, with concepts and methods which relied on algebra and differential and integral calculus, these terms being taken into account in the sense they were used at that time. Furthermore, scientific thought was fundamentally deterministic during this period, even if the existence of games of chance using mechanical devices (dice, roulette, etc.) seemed far from the philosophy of science or Cauchy’s theorem. Each time has
its concepts, which are based on the current state of knowledge, and the science of fluid mechanics was reduced for the most part to semi-empirical engineering formulae and to particular analytical solutions. Between the 1920s and the 1950s, our ideas on boundary layers and hydrodynamic stability were progressively elucidated. Studies of turbulence, which began in the 1920s from a conceptual statistical point of view, have really only made further progress in the 1970s, with the writing of the balance equations using turbulence models with a physical basis. This progress remains quite modest, however, considering the immensity of the task which remains.

It should be noted that certain disciplines have seen a spectacular renewal since the 1970s for two main reasons: on the one hand, the development of information technology has provided formidable computation and experimental methods, and on the other hand, multidisciplinary problems have arisen from industrial necessities. Acoustics is a typical example: many problems of propagation had been solved in the 1950s-1960s and those which were not made only very slow progress. Physics focused on other fundamental, more promising sectors (semiconductors, properties of matter, etc.). However, in the face of a need to provide practical solutions to industrial problems (sound generated by fluid flow, the development of ultra-sound equipment, etc.), acoustics became an engineering science in the 1970s. Acoustics is indeed a domain of compressible fluid mechanics and it will constitute an integral part of our treatment of the subject.

Parallel to this, systems became an object of study in themselves (automatic control) and the possibilities of study and understanding of the complexity progressed (signal processing, modeling of systems with large numbers of variables, etc.). Determinism itself is now seen in a more modest light: it suffices to remember the variable level of our ambitions with regard to meteorological prediction in the last 30 years to see that we have not yet arrived at a point where we have a definite set of concepts. Meteorological phenomena are largely governed by fluid mechanics.

The conception of this book results from the preceding observations. The author refuses to get into the argument which consists of saying that the time of analytical solutions has passed and that numerical simulation will solve all our problems. The reality is clearly more subtle than this: analytical solution in the broad sense, that is, the obtaining of results derived from reasoning and mathematical concepts, is the basis of physical concepts. Computations performed by computers by themselves cannot provide any more insight than an experiment, although both must be performed with great care. The state of knowledge and of understanding of mechanisms varies depending on the domain studied. In particular, the science of turbulence is still at a somewhat embryonic stage, and the mystery of turbulent solutions of the Navier-Stokes equations is far from being thoroughly cleared up.
We are still at the stage of Galileo who attempted to understand mechanics without the ideas of differential calculus. Nobody can today say precisely what are the difficulties to be solved, and the time which will be required for their resolution (10 years, a century or 10 centuries). We will therefore present the state of our knowledge in the current scientific context by also considering some of the accompanying disciplines (thermodynamics, ideas related to partial differential equations, signal processing, system analysis) which are directly useful to the concepts, modeling, experiments and applications in fluid mechanics and energetics of flows. We will not cover specific combustion phenomena, limiting ourselves to a few simplified cases of physico-chemical reactions.

This book covers the necessary fundamentals for the study and understanding of the specific concepts and general properties of flows: the establishment and discussion of the balance equations of extensive quantities in fluid motions, the transport of these quantities by convection, wave-propagation or diffusion. These physical concepts are issued from the comprehension of theoretical notions associated with equations, such as characteristic curves or surfaces, perturbation methods, modal developments (Fourier series, etc.) and integral transforms, model reduction, etc. These mathematical aspects are either consequences of properties of partial differential equations or derived from other disciplines such as signal processing and system analysis, whose impact is important in every scientific or technological domain. They are discussed and illustrated by some elementary problems of fluid mechanics and thermal conduction, including measurement methods and experimental data processing This book is an introduction to the study of more specialized topics of fluid flow and transfer phenomena encountered in different domains of application: incompressible or compressible flow, dynamic and thermal boundary layers, natural or mixed convection, 3D boundary layers, physico-chemical reactions in flows, acoustics in flows, aerodynamic sound, thermoacoustics, etc.

Chapter 1 is devoted to a synthetic presentation of thermodynamics. After recalling the basics of the representation of material systems, thermostatics is covered in an axiomatic fashion which avoids the use of differential formulations and which allows for a simplified presentation of classical results. Taking entropy dynamics as a starting point, the thermodynamics of non-equilibrium states is then discussed using simple examples with phenomenological laws of linear thermodynamics.

The continuous medium at rest is obtained by taking the limit of discrete systems in Chapter 2. The exchange of extensive quantities is modeled by means of flux densities, and irreversible thermodynamics leads to the diffusion equations. Some reminders of fluid statics are given. We then discuss the difficulties specific to the diffusion of matter.