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Edited by
Fan Lianghuo • Wong Ngai-Ying
Cai Jinfta • Li Shiqi

HOW CHINESE LEARN MATHEMATICS
Perspectives from Insiders
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SERIES ON MATHEMATICS EDUCATION

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Focusing on the Chinese Way of Learning Mathematics: An Introduction

The last decades have seen a mounting interest in Asian education amongst politicians, policy makers, educational researchers and practitioners around the world. In particular, being part of Asian education, Chinese education has attracted much attention. This growth of interest is clearly relevant to the wide acclaimed performance of Asian and particularly Chinese students that people have observed in large-scale international comparisons, such as those conducted in the International Assessment of Educational Progress (IAEP), the Third International Mathematics and Science Study (TIMSS), and the Programme for International Student Assessment (PISA) during this period. In those comparisons, mathematics as a school subject has been a focus because of its importance in children’s school education and its comparability in contents being taught in different countries. Needless to say, many questions in this regard remain to be studied, or even identified.

This book intends to present a concerted effort in the study of Chinese mathematics education by a group of international researchers, both Chinese and non-Chinese, who we believe have insiders’ experience, expertise, and more importantly a passion concerning Chinese mathematics education. The focus of the book is on how Chinese learn mathematics. Given the long history of Chinese culture, the rich practice of Chinese pedagogy, and the large population of Chinese learners, the value of studying on Chinese mathematics education to itself and beyond is easy to see.
The origin of the book can be traced back to the Ninth International Congress on Mathematical Education (ICME-9) held in Tokyo, Japan in 2000, during which a special event of the congress, *Forum of All Chinese Math Educators: Festive and Academic Gathering of Chinese Descendants and Other Interested Parties*, drew much interest from an unexpectedly large number of audience from Mainland China, Hong Kong, Singapore, Taiwan, Russia, Japan, and the United States, etc., in its two sessions. With the momentum and encouragement we gained from the event and all the interested scholars and researchers including particularly the four advisors of this book, we started our journey of more than three years, from the initial discussion of the main theme and structure to the organization of peer-reviewing for all the contributions, and finally to the completion of the book.

This book consists of four sections. In **Section 1, “Overview and International Perspectives”**, we start off in Chapter 1 with a comprehensive overview and synthesis by Fan and Zhu about the performance of Chinese students as revealed in large-scale international comparative studies in mathematics education conducted over the past few decades. The overview is followed by Wong, Han, and Lee’s investigation of current mathematics curricula in 10 countries/regions in Chapter 2, providing readers with a broad background from an international and comparative perspective to understand the curriculum environment, under which Chinese students learn mathematics. In Chapter 3, Cai and Cifarelli provided an insightful review of smaller-scale and more specific cross-national comparative studies of the US and Chinese students and presented a profile of Chinese learners’ mathematical thinking in problem solving and problem posing. In particular, they identified six characteristics of Chinese learners’ mathematical thinking in problem solving. Following that, Wang and Murphy further led us in Chapter 4 into the Chinese classroom by analyzing discourse in a model Chinese mathematics classroom and developing the concept of coherence to describe the most salient

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features found in the classroom, that is, coherence in the instructional, psychological, and social dimensions. In Chapter 5, Li Jin in a more generic meaning proposed a Chinese cultural model of learning, based on her examination of Chinese learners’ beliefs with respect to the learning purposes, processes, achievement standards, and affect by contrasting with the American ones. Meanwhile, Siu in Chapter 6 further opened up the cultural root of the Chinese mathematics learners by a uniquely “animated” reflection from a historical perspective on the official mathematics curriculum and the state examination system in ancient China. The chapter showed that study in mathematics in ancient China did not proceed in an examination-oriented, rote-based learning environment. In sum, by looking into the Chinese mathematics learners with different glass (telescopic, microscopic and longitudinal), we hope a general image of the Chinese mathematics learners is portrayed in Section 1.

Section 2, “Context and Teaching Materials”, examines the social and pedagogical context and teaching materials particularly textbooks with the focus on how they affect the way Chinese students learn mathematics in classrooms, in schools, and in their own homes. In Chapter 7, Zhang, Li Shiqi, and Tang analyzed a once most coherent and visible principle for mathematics instruction in Mainland China, the “two basics” principle: “basic knowledge and basic skills”, and discussed its both positive and negative influences on Chinese students’ learning of mathematics. Bao in Chapter 8 evaluated the difficulty levels of new and old mathematics textbooks measured in five dimensions and provided us with an updated view of the mathematics curriculum in Mainland China. Also focusing on the mathematics textbooks, Fan, Chen, Zhu, Qiu, and Hu investigated in Chapter 9 the role of textbooks in the process of Chinese students’ learning of mathematics through looking into how they are used by teachers and students within and beyond classrooms in two Chinese cities, Kuming and Fuzhou, providing readers with insights on how textbooks as main teaching materials shape the way of teaching and learning. Differently, Li Jianhua in Chapter 10 focused on a popular Chinese elementary school mathematics teacher manual in the topic of multi-digit multiplication, and illuminated a significant feature of the Chinese teachers’ manual, that is, offering a thorough understanding of
the student text and detailed suggestion for teachers’ teaching. In Chapter 11, Huang Hsin Mei examined the effects of cram schools in Taiwan on children’s mathematics learning. As the influence of non-formal mathematics education students received outside schools on their achievement particularly in Asia societies, such as Singapore, Hong Kong, Taiwan, Japan and Mainland China, have received growing attention in international mathematics education community, and yet they are still under-researched, this chapter opened a fresh and meaningful window for us to view mathematics instruction beyond the regular school settings.

Section 3, “Pedagogy and Learning Processes”, focuses on the general pedagogy and learning processes in the Chinese mathematics classrooms. It begins with an analysis in Chapter 12 by Gu, Huang Rongjin, and Marton on how Chinese teachers promote effective mathematics learning even with large classes by teaching with variation, a characteristic they identified in Chinese mathematics classroom and a theory they have been working first independently then collaboratively on mathematics pedagogy. Still with the perspective of variations, Huang Rongjin and Leung in Chapter 13 further looked into the Hong Kong and Shanghai classroom, and their findings challenged the very idea of the so-called paradox of Chinese learners. Similarly taking a Shanghai classroom as a focal point of investigation, Lopez-Real, Mok, Leung, and Marton revealed in Chapter 14 that although one might be able to identify a “pattern” of teaching that characterizes a teacher’s approach, it is dangerous to attempt to look for a national “script” of mathematics lessons. Consistently, Ma, Zhao, and Tuo further in Chapter 15 alerted us to the diversity within the perceived communality of the so-called “Chinese” mathematics classrooms by showing a vast difference in teachers and their preparation and implementation of teaching in rural and urban regions in Northeast China. In Chapter 16, Li Jun examined the learning results of using both theoretical and experimental

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2 For example, see the Discussion Document of ICMI Study 13, “Mathematics education in different cultural traditions: A comparative study of East Asia and the West”, retrievable at its official website at http://www.inf.fu-berlin.de/icmics (as of 15 May 2004).
approaches in teaching a specific topic of mathematics curriculum, probability, on Chinese students, and hence revealed the advantages and disadvantages of both approaches. Using date collected from mathematics classrooms in Jiangsu Province of China, An in Chapter 17 identified and discussed a unique characteristic of Chinese mathematics teaching and learning: using the learning-questioning and learning-reviewing instructional model to enhance students’ understanding of mathematics concepts and reinforce mathematics proficiency. Finally, Xu in Chapter 18 reported a study on the effects of using different representations in problem solving activities on Chinese children’s learning of mathematics, and concluded that by means of different representations the Chinese children could exhibit their different preferred cognitive structures, which signals a challenge for teachers to change their conception about teaching and learning.

Section 4, “Inspiration and Future Directions”, presents the last two chapters of the book. In Chapter 19, Wong firstly provided a comprehensive review on the trend of CHC (Confucian Heritage Culture) studies over the past fifteen years, and then examined the three Chinese traditions of calligraphy, martial art, and seal carving, and their implications for our understanding the learning of mathematics, and thus finally provided his perception of the “CHC script” concerning our main theme. In Chapter 20, Cai, Lin, and Fan offered a summary of research-based evidences that characterize how Chinese learn mathematics around the following four key issues: (1) Are Chinese learners really higher achievers in mathematics? (2) Does the teaching of Chinese learners necessarily lead to rote learning? (3) How is the intended curriculum structured to support teaching and learning? and (4) How do Chinese families support students’ learning? Furthermore, for each of these questions, they discussed needed directions to better understand Chinese learners and their learning of mathematics, and thus closed the whole book.

This book could not have been completed without many people’s help, support, and cooperation. As editors, first we would like to thank our advisors, Zhang Dianzhou of East China Normal University, Lee Peng Yee of the National Institute of Education (NIE) of Nanyang Technological University, Lin Fou-Lai of National Taiwan Normal
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It is clear to us that, in many senses, this book only represents a starting point in our understanding the phenomenon and exploring the value of mathematics education involving Chinese students, and there is much to do further along this direction. In this connection, we hope the book can contribute meaningfully in the long way to the advancement of research in Chinese mathematics education, and hence more broadly in the international mathematics education.

Fan Lianghuo
Wong Ngai-Ying
Cai Jinfa
Li Shiqi

May 2004
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