Statistical Models and Methods for Lifetime Data

Second Edition

JERALD F. LAWLESS
University of Waterloo
This page intentionally left blank
Statistical Models and Methods
for Lifetime Data
To Liz
This page intentionally left blank
# Contents

Preface to Second Edition
Preface to First Edition

1. **Basic Concepts and Models**
   1.1 Introduction, 1
   1.2 Lifetime Distributions, 8
      1.2.1 Continuous Models, 8
      1.2.2 Discrete Models, 10
      1.2.3 A General Formulation, 11
      1.2.4 Some Remarks on the Hazard Function, 14
   1.3 Some Important Models, 16
      1.3.1 The Exponential Distribution, 17
      1.3.2 The Weibull Distribution, 18
      1.3.3 The Log-Normal Distribution, 21
      1.3.4 The Log-Logistic Distribution, 23
      1.3.5 The Gamma Distribution, 25
      1.3.6 Log-Location-Scale Models, 26
      1.3.7 The Inverse Gaussian Distribution, 30
      1.3.8 Models with Piecewise Constant or Polynomial Hazard Functions, 30
      1.3.9 Some Other Models, 33
      1.3.10 Mixture Models, 33
   1.4 Regression Models, 34
   1.5 Multiple Lifetimes and Multiple Modes of Failure, 36
   1.6 Some Comments on Model Selection and Statistical Analysis, 37

Bibliographic Notes, 39
Computational Notes, 40
Problems and Supplements, 41
2. Observation Schemes, Censoring, and Likelihood

2.1 Introduction, 49

2.2 Right Censoring and Maximum Likelihood, 52
  2.2.1 Some Types of Right Censoring, 52
  2.2.2 A General Formulation of Right Censoring, 58
  2.2.3 Likelihood Inference with Censored Data, 61

2.3 Other Types of Incomplete Data, 63
  2.3.1 Intermittent Observation and Interval Censoring, 63
  2.3.2 Double Censoring, 66
  2.3.3 Remarks on Missing Data, 66

2.4 Truncation and Selection Effects, 67
  2.4.1 Delayed Entry and Left Truncation, 67
  2.4.2 Retrospective Observation and Selection Effects, 70

2.5 Information and Design Issues, 71

Bibliographic Notes, 74

Problems and Supplements, 75

3. Some Nonparametric and Graphical Procedures

3.1 Introduction, 79

3.2 Nonparametric Estimation of a Survivor Function and Quantiles, 80
  3.2.1 The Product-Limit Estimate, 80
  3.2.2 The Nelson–Aalen Estimate, 85
  3.2.3 Interval Estimation of Survival Probabilities or Quantiles, 87
  3.2.4 Asymptotic Properties of Estimators, 95

3.3 Descriptive and Diagnostic Plots, 98
  3.3.1 Plots Involving Survivor or Cumulative Hazard Functions, 98
  3.3.2 Classic Probability Plots, 108

3.4 Estimation of Hazard or Density Functions, 109
  3.4.1 General Remarks, 109
  3.4.2 Some Simple Procedures and an Example, 111

3.5 Methods for Truncated and Interval Censored Data, 115
  3.5.1 Left-Truncated Data, 116
  3.5.2 Right-Truncated Data, 120
  3.5.3 Interval-Censored Data, 124
3.6 Life Tables, 128
   3.6.1 Standard Life Table Methods, 128
   3.6.2 Theory for Life Table Methodology, 132

Bibliographic Notes, 136
Computational Notes, 138
Problems and Supplements, 139

4. Inference Procedures for Parametric Models 147
   4.1 Inference Procedures for Exponential Distributions, 147
      4.1.1 Methods Based on Large-Sample Theory, 148
      4.1.2 Exact Methods for Certain Life Tests, 152
      4.1.3 Comparison of Distributions, 154
      4.1.4 Planning Experiments or Life Tests, 157
   4.2 Inference Procedures for Some Other Models, 164
      4.2.1 The Gamma Distribution, 164
      4.2.2 The Inverse Gaussian Distribution, 170
      4.2.3 Models with Polynomial-Based Hazard Functions, 172
   4.3 Grouped, Interval Censored, or Truncated Data, 174
      4.3.1 Grouped Lifetimes, 174
      4.3.2 Interval-Censored Data, 176
      4.3.3 Truncated Data, 177
   4.4 Mixture Models, 181
   4.5 Threshold Parameters, 185
      4.5.1 General Remarks, 186
      4.5.2 The Three-Parameter Weibull Distribution, 187
      4.5.3 The Two-Parameter Exponential Distribution, 190
   4.6 Prediction Intervals, 194

Bibliographic Notes, 200
Computational Notes, 201
Problems and Supplements, 202

5. Inference Procedures for Log-Location-Scale Distributions 211
   5.1 Inference for Location-Scale Distributions, 211
      5.1.1 Likelihood-Based Methods, 212
      5.1.2 Exact Procedures Under Type 2 Censoring, 217
   5.2 Weibull and Extreme-Value Distributions, 218
      5.2.1 Likelihood-Based Inference Procedures, 218
      5.2.2 Exact Confidence Intervals Under Type 2 Censoring, 223
   5.3 Log-Normal and Log-Logistic Distributions, 230
5.3.1 Inferences for Log-Normal and Normal Models, 230
5.3.2 Inferences for Log-Logistic and Logistic Models, 231
5.3.3 Examples, 232
5.4 Comparison of Distributions, 235
5.4.1 General Methods for Comparing (Log-)Location-Scale Distributions, 236
5.4.2 Comparison of Weibull or Extreme Value Distributions, 238
5.4.3 Comparison of Log-Normal or Log-Logistic Distributions, 242
5.5 Models with Additional Shape Parameters, 243
5.5.1 Introduction, 243
5.5.2 The Generalized Log-Burr Distribution, 244
5.5.3 The Generalized Log-Gamma Distribution, 247
5.6 Planning Experiments or Life Tests, 252
5.6.1 Information Calculations Under Type 1 or Type 2 Censoring, 252
5.6.2 Formal Tests and Acceptance Procedures, 253
5.6.3 An Example, 255
Bibliographic Notes, 257
Computational Notes, 258
Problems and Supplements, 258

6. Parametric Regression Models 269
6.1 Introduction, 269
6.1.1 Log-Location-Scale (Accelerated Failure Time) Regression Models, 270
6.1.2 Proportional Hazards Regression Models, 271
6.1.3 Other Regression Models, 273
6.2 Graphical Methods and Model Assessment, 274
6.2.1 Looking for Models, 274
6.2.2 Assessment of Fitted Models, 283
6.3 Inference for Log-Location-Scale (Accelerated Failure Time) Models, 292
6.3.1 Likelihood Methods, 292
6.3.2 Weibull and Extreme Value Regression Models, 296
6.3.3 Normal-Log-Normal and Logistic-Log-Logistic Regression Models, 303
6.3.4 Some Comments on Least Squares, Robustness, and Efficiency, 306
6.3.5 Experimental Design, 308
6.4 Extensions of Log-Location-Scale Models, 311
   6.4.1 Families of Error Distributions, 311
   6.4.2 Variable Scale Parameters, 317
   6.4.3 Time-Varying Covariates, 320

6.5 Some Other Models, 321
   6.5.1 Hazard-Based Models, 322
   6.5.2 Mechanistic Models, 325
   6.5.3 Transformations and Some Other Models, 327

Bibliographic Notes, 329
Computational Notes, 330
Problems and Supplements, 331

7. Semiparametric Multiplicative Hazards Regression Models 341

   7.1 Methods for Continuous Multiplicative Hazards Models, 341
      7.1.1 Estimation and Tests for $\beta$, 342
      7.1.2 Comparison of Two or More Lifetime Distributions, 344
      7.1.3 Justification and Properties of the Likelihood Function $L(\beta)$, 348
      7.1.4 Adjustments for Tied Lifetimes, 351
      7.1.5 Estimation of $H_0(t)$ or $S_0(t)$, 352
      7.1.6 Stratification, 354
      7.1.7 Left Truncation and Delayed Entry, 355
      7.1.8 Time-Varying Covariates, 355
      7.1.9 Model Checking, 358

   7.2 Examples, 363

   7.3 Methods for Grouped or Discrete Lifetimes, 370
      7.3.1 Regression Analysis of Grouped Lifetimes, 370
      7.3.2 Testing the Equality of Distributions with Grouped Data, 376
      7.3.3 Discrete-Time Hazard-Based Models, 382

   7.4 Semiparametric Maximum Likelihood, 384
      7.4.1 Estimation from Continuous Observation, 384
      7.4.2 Estimation from Incomplete Data, 387

Bibliographic Notes, 388
Computational Notes, 390
Problems and Supplements, 390
8. Rank-Type and Other Semiparametric Procedures for Log-Location-Scale Models 401

8.1 Rank Tests for Comparing Distributions, 402
   8.1.1 Linear Rank Tests for the m-Sample Problem, 402
   8.1.2 The Exponential Ordered Scores (Log Rank) Test with Censored Data, 409
   8.1.3 The Generalized Wilcoxon Test with Censored Data, 411
   8.1.4 Counting Process Formulation of m-Sample Tests, 413
   8.1.5 Discussion and Examples, 416

8.2 Estimation for Semiparametric AFT Models, 420
   8.2.1 Rank-Based Procedures with Fixed Covariates, 420
   8.2.2 Rank-Type Procedures with Time-Varying Covariates, 424
   8.2.3 Discussion, 426

Bibliographic Notes, 428
Computational Notes, 429
Problems and Supplements, 429

9. Multiple Modes of Failure 433

9.1 Introduction, 433
   9.1.1 Basic Characteristics and Model Specification, 433
   9.1.2 Likelihood Function Formulation, 435

9.2 Nonparametric Methods, 437
9.3 Parametric Methods, 444
   9.3.1 Grouped or Discrete Data, 448
9.4 Semiparametric Methods for Multiplicative Hazards Models, 449
   9.4.1 Estimation of Cumulative Incidence Functions, 452

Bibliographic Notes, 455
Problems and Supplements, 456

10. Goodness-of-Fit Tests 465

10.1 Introduction, 465
10.2 Some General Methods of Testing Fit, 467
   10.2.1 Tests Based on Comparing Empirical and Hypothesized Distributions, 467
   10.2.2 Model Expansion and Smooth Tests, 469
   10.2.3 Tests Based on Grouped Data, 471
10.3 Tests of Fit for Specific Distributions, 476
10.3.1 Tests of Fit for the Exponential Distribution, 476
10.3.2 Tests of Fit for the Weibull and Extreme Value Distributions, 478
10.3.3 Tests of Fit for the Normal and Log-Normal Distributions, 480
10.3.4 Additional Remarks, 481
10.4 Tests of Fit with Regression Models, 482
  10.4.1 General Remarks, 482
  10.4.2 Location-Scale Regression Models, 483
  10.4.3 Multiplicative Hazards Models, 484

Bibliographic Notes, 485
Problems and Supplements, 486

11. Beyond Univariate Survival Analysis 491
  11.1 Introduction, 491
  11.2 Multivariate Lifetime Distributions and Clustered Lifetimes, 493
    11.2.1 Multivariate Lifetime Distributions, 494
    11.2.2 Maximum Likelihood and Pseudolikelihood Methods, 500
    11.2.3 An Example, 504
  11.3 Sequences of Lifetimes, 507
    11.3.1 Some Models and Methods, 507
    11.3.2 An Example, 509
  11.4 General Event History Processes, 512
  11.5 Failure and Related Processes, 518
    11.5.1 Some Context and Objectives, 518
    11.5.2 Some Approaches to Modeling and Analysis, 520
Bibliographic Notes, 523
Computational Notes, 525
Problems and Supplements, 526

Appendices
A. Glossary of Notation and Abbreviations 535
  A.1 Notation and Symbols, 535
  A.2 Abbreviations, 536
B. Asymptotic Variance Formulas, Gamma Functions, and Order Statistics 539
  B.1 Asymptotic Variance Formulas, 539
  B.2 Gamma Functions, 541
  B.3 Order Statistics, 542