Multi-moment Asset Allocation
and Pricing Models
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Multi-moment Asset Allocation 
and Pricing Models

Edited by

Emmanuel Jurczenko and Bertrand Maillet

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To our wives and children
and specifically to Manon and Viktor:
welcome to our skewed and leptokurtic world . . .
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Emmanuel Jurczenko and Bertrand Maillet

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