Introduction to Optical Waveguide Analysis

Solving Maxwell’s Equations and the Schrödinger Equation

KENJI KAWANO
TSUTOMU KITOH

WILEY
INTRODUCTION TO OPTICAL WAVEGUIDE ANALYSIS
This page intentionally left blank
INTRODUCTION TO OPTICAL WAVEGUIDE ANALYSIS
Solving Maxwell’s Equations and the Schrödinger Equation

KENJI KAWANO and TSUTOMU KITOH

A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.
New York / Chichester / Weinheim / Brisbane / Singapore / Toronto
To our wives,
Mariko and Kumiko
This page intentionally left blank
CONTENTS

Preface / xi

1 Fundamental Equations 1

1.1 Maxwell's Equations / 1
1.2 Wave Equations / 3
1.3 Poynting Vectors / 7
1.4 Boundary Conditions for Electromagnetic Fields / 9

Problems / 10
Reference / 12

2 Analytical Methods 13

2.1 Method for a Three-Layer Slab Optical Waveguide / 13
2.2 Effective Index Method / 20
2.3 Marcatili's Method / 23
2.4 Method for an Optical Fiber / 36

Problems / 55
References / 57
3 Finite-Element Methods

3.1 Variational Method / 59
3.2 Galerkin Method / 68
3.3 Area Coordinates and Triangular Elements / 72
3.4 Derivation of Eigenvalue Matrix Equations / 84
3.5 Matrix Elements / 89
3.6 Programming / 105
3.7 Boundary Conditions / 110
Problems / 113
References / 115

4 Finite-Difference Methods

4.1 Finite-Difference Approximations / 118
4.2 Wave Equations / 120
4.3 Finite-Difference Expressions of Wave Equations / 127
4.4 Programming / 150
4.5 Boundary Conditions / 153
4.6 Numerical Example / 160
Problems / 161
References / 164

5 Beam Propagation Methods

5.1 Fast Fourier Transform Beam Propagation Method / 165
5.2 Finite-Difference Beam Propagation Method / 180
5.3 Wide-Angle Analysis Using Padé Approximant Operators / 204
5.4 Three-Dimensional Semivectorial Analysis / 216
5.5 Three-Dimensional Fully Vectorial Analysis / 222
Problems / 227
References / 230

6 Finite-Difference Time-Domain Method

6.1 Discretization of Electromagnetic Fields / 233
6.2 Stability Condition / 239
6.3 Absorbing Boundary Conditions / 241
CONTENTS

Problems / 245
References / 249

7 Schrödinger Equation / 251

7.1 Time-Dependent State / 251
7.2 Finite-Difference Analysis of Time-Independent State / 253
7.3 Finite-Element Analysis of Time-Independent State / 254
References / 263

Appendix A Vectorial Formulas / 265

Appendix B Integration Formula for Area Coordinates / 267

Index / 273
This book was originally published in Japanese in October 1998 with the intention of providing a straightforward presentation of the sophisticated techniques used in optical waveguide analyses. Apparently, we were successful because the Japanese version has been well accepted by students in undergraduate, postgraduate, and Ph.D. courses as well as by researchers at universities and colleges and by researchers and engineers in the private sector of the optoelectronics field. Since we did not want to change the fundamental presentation of the original, this English version is, except for the newly added optical fiber analyses and problems, essentially a direct translation of the Japanese version.

Optical waveguide devices already play important roles in telecommunications systems, and their importance will certainly grow in the future. People considering which computer programs to use when designing optical waveguide devices have two choices: develop their own or use those available on the market. A thorough understanding of optical waveguide analysis is, of course, indispensable if we are to develop our own programs. And computer-aided design (CAD) software for optical waveguides is available on the market. The CAD software can be used more effectively by designers who understand the features of each analysis method. Furthermore, an understanding of the wave equations and how they are solved helps us understand the optical waveguides themselves.

Since each analysis method has its own features, different methods are required for different targets. Thus, several kinds of analysis methods have
to be mastered. Writing formal programs based on equations is risky unless one knows the approximations used in deriving those equations, the errors due to those approximations, and the stability of the solutions.

Mastering several kinds of analysis techniques in a short time is difficult not only for beginners but also for busy researchers and engineers. Indeed, it was when we found ourselves devoting substantial effort to mastering various analysis techniques while at the same time designing, fabricating, and measuring optical waveguide devices that we saw the need for an easy-to-understand presentation of analysis techniques.

This book is intended to guide the reader to a comprehensive understanding of optical waveguide analyses through self-study. It is important to note that the intermediate processes in the mathematical manipulations have not been omitted. The manipulations presented here are very detailed so that they can be easily understood by readers who are not familiar with them. Furthermore, the errors and stabilities of the solutions are discussed as clearly and concisely as possible. Someone using this book as a reference should be able to understand the papers in the field, develop programs, and even improve the conventional optical waveguide theories.

Which optical waveguide analyses should be mastered is also an important consideration. Methods touted as superior have sometimes proven to be inadequate with regard to their accuracy, the stability of their solutions, and central processing unit (CPU) time they require. The methods discussed in this book are ones widely accepted around the world. Using them, we have developed programs we use on a daily basis in our laboratories and confirmed their accuracy, stability, and effectiveness in terms of CPU time.

This book treats both analytical methods and numerical methods. Chapter 1 summarizes Maxwell’s equations, vectorial wave equations, and the boundary conditions for electromagnetic fields. Chapter 2 discusses the analysis of a three-layer slab optical waveguide, the effective index method, Marcatili’s method, and the analysis of an optical fiber. Chapter 3 explains the widely utilized scalar finite-element method. It first discusses its basic theory and then derives the matrix elements in the eigenvalue equation and explains how their calculation can be programmed. Chapter 4 discusses the semivectorial finite-difference method. It derives the fully vectorial and semivectorial wave equations, discusses their relations, and then derives explicit expressions for the quasi-TE and quasi-TM modes. It shows formulations of $E_x$ and $H_y$ expressions for the quasi-TE (transverse electric) mode and $E_y$ and $H_x$ expressions for the quasi-TM (transverse magnetic) mode. The none-
quidistant discretization scheme used in this chapter is more versatile than
the equidistant discretization reported by Stern. The discretization errors
due to these formulations are also discussed. Chapter 5 discusses beam
propagation methods for the design of two- and three-dimensional (2D,
3D) optical waveguides. Discussed here are the fast Fourier transform
beam propagation method (FFT-BPM), the finite-difference beam propa-
gation method (FD-BPM), the transparent boundary conditions, the wide-
gle FD-BPM using the Padé approximant operators, the 3D semi-
vectorial analysis based on the alternate-direction implicit method, and
the fully vectorial analysis. The concepts of these methods are discussed
in detail and their equations are derived. Also discussed are the error
factors of the FFT-BPM, the physical meaning of the Fresnel equation,
the problems with the wide-angle FFT-BPM, and the stability of the
FD-BPM. Chapter 6 discusses the finite-difference time-domain method
(FD-TDM). The FD-TDM is a little difficult to apply to 3D optical
waveguides from the viewpoint of computer memory and CPU time, but
it is an important analysis method and is applicable to 2D structures.
Covered in this chapter are the Yee lattice, explicit 3D difference
formulation, and absorbing boundary conditions. Quantum wells, which
are indispensable in semiconductor optoelectronic devices, cannot be
designed without solving the Schrödinger equation. Chapter 7 discusses
how to solve the Schrödinger equation with the effective mass approx-
imation. Since the structure of the Schrödinger equation is the same as that
of the optical wave equation, the techniques to solve the optical wave
equation can be used to solve the Schrödinger equation.

Space is saved by including only a few examples in this book. The
quasi-TEM and hybrid-mode analyses for the electrodes of microwave
integrated circuits and optical devices have also been omitted because of
space limitations. Finally, we should mention that readers are able to get
information on the vendors that provide CAD software for the numerical
methods discussed in this book from the Internet.

We hope this book will help people who want to master optical
waveguide analyses and will facilitate optoelectronics research and devel-
opment.

KENJI KAWANO and TSUTOMU KITOH

Kanagawa, Japan
March 2001