This book is intended for Undergraduate and First year Post graduate students in the disciplines of Mechanical and Aerospace engineering. Also, scientists and engineers working in the areas of aerospace propulsion and gas dynamics should find this book to be a valuable addition to their collection of books on the subject matter. This edition of the book incorporates changes in the development of the material, new material and figures as well as more end of chapter problems in all the chapter.

The unique feature of this edition is the addition of a chapter on the gas dynamics of the flow of steam in nozzles. In keeping with the spirit of the first edition, the worked examples and exercise problems have almost all been drawn from practical applications in propulsion and gas dynamics. These are comprehensive and are formulated to test the understanding of the subject matter and thus serve as a self-check for the reader.

Introduction • One Dimensional Flows – Basics • Normal Shock Waves • Flow with Heat Addition- Rayleigh Flow • Flow with Friction - Fanno Flow • Quasi One Dimensional Flows • Oblique Shock Waves • Prandtl Meyer Flow • Flow of Steam through Nozzles • Exercises • Suggested Reading • Index

Dr. V. Babu is currently an Associate Professor in the Department of Mechanical Engineering at IIT Madras. He received his B.E. in Mechanical Engineering from REC Trichy in 1985 and Ph. D from the Ohio State University in 1991. He worked as a Post-Doctoral researcher at the Ohio State University from 1991 to 1995. He was a Technical Specialist at the Ford Scientific Research Lab, Dearborn, Michigan from 1995 to 1998. He joined IIT Madras at the end of 1998. He received the Henry Ford Technology Award in 1998 for the development and deployment of a virtual wind tunnel. He has four U.S. patents to his credit. He has published technical papers on simulations of fluid flows including plasmas and non-equilibrium flows, computational aerodynamics and aeroacoustics, scientific computing and ramjet, scramjet engines. His primary research specialization is CFD and he is currently involved in the simulation of high speed reacting flows, prediction of jet noise, simulation of fluid flows using the lattice Boltzmann method and high performance computing.
Fundamentals of Gas Dynamics

(2nd Edition)
Cover illustration: Schlieren picture of an under-expanded flow issuing from a convergent divergent nozzle. Prandtl-Meyer expansion waves in the divergent portion as the flow goes around the convex throat can be seen. Expansion fans, reflected oblique shocks and the alternate swelling and compression of the jet are clearly visible. Courtesy: P. K. Shijin, PhD scholar, Dept. of Mechanical Eng, IIT Madras.
Fundamentals of Gas Dynamics

(2nd Edition)

V. Babu
Professor
Department of Mechanical Engineering
Indian Institute of Technology, Madras, INDIA

WILEY
John Wiley & Sons Ltd.

Athena Academic Ltd.
Dedicated to my wife
Chitra and son Aravindh
for their enduring
patience and love
I am happy to come out with this edition of the book *Fundamentals of Gas Dynamics*. Readers of the first edition should be able to see changes in all the chapters - changes in the development of the material, new material and figures as well as more end of chapter problems. In keeping with the spirit of the first edition, the additional exercise problems are drawn from practical applications to enable the student to make the connection from concept to application.

Owing to the ubiquitous nature of steam power plants around the world, it is important for mechanical engineering students to learn the gas dynamics of steam. With this in mind, a new chapter on the gas dynamics of steam has been added in this edition. This is somewhat unusual since this topic is usually introduced in text books on steam turbines and not in gas dynamics texts. In my opinion, introducing this in a gas dynamics text is logical and in fact makes it easy for the students to learn the concepts. In developing this material, I have assumed that the reader would have gone through a fundamental course in thermodynamics and so would be familiar with calculations involving steam. Steam tables for use in these calculations have also been added at the end of the book. I would like to thank Prof. Korpela of the Ohio State University for generating these tables and allowing me to include them in the book.
I wish to thank the readers who purchased the first edition and gave me many suggestions as well as for pointing out errors. To the extent possible, the errors have been corrected and the suggestions have been incorporated in this edition. If there are any errors or if you have any suggestions for improving the exposition of any topic, please feel free to communicate them to me via e-mail (vbabu@iitm.ac.in). I would like to take this opportunity to thank Prof. S. R. Chakravarthy of IIT Madras for his suggestion concerning the definition of compressibility. I have taken this further and connected it with Rayleigh flow in the incompressible limit. The effect of different $\gamma$ on the property changes across a normal shock wave are now included in Chapter 3. The development of the process curve in Chapters 4 and 5 has been done by directly relating the changes in properties to changes in stagnation temperature and entropy respectively. In Chapter 6, I have added a figure showing the variation of static pressure along a CD nozzle as well as the variation of exit static pressure to the ambient pressure. Hopefully this will make it easier for the student to understand over- and under-expanded flow.

Once again I would like to express my heartfelt gratitude to my teachers who taught me so much without expecting anything in return. I can only hope that I succeed in giving back at least a fraction of the knowledge and wisdom that I received from them. My advisor, mentor and friend, Prof. Seppo Korpela has been an inspiration to me and his constant and patient counsel has helped me enormously. I am indebted to my parents for the sacrifices they made to impart a good education to me. This is not a debt that can be repaid. But for the constant support and encouragement from my wife and son, this edition and the other books that I have written would not have been possible.

Finally, I would like to thank my former students P. S. Tide, S. Somasundaram and Anandraj Hariharan for diligently working out the examples and exercise problems and my current student P. K. Shijin for carefully proof reading the manuscript and making helpful suggestions. Thanks are due in addition to Prof. P. S. Tide for preparing the Solutions Manual.

V. Babu
# CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td></td>
<td>vii</td>
</tr>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.1 Compressibility of Fluids</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 Compressible and Incompressible Flows</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1.3 Perfect Gas Equation of State</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1.3.1 Continuum Hypothesis</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1.4 Calorically Perfect Gas</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>One Dimensional Flows - Basics</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2.1 Governing Equations</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2.2 Acoustic Wave Propagation Speed</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>2.2.1 Mach Number</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2.3 Reference States</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.3.1 Sonic State</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.3.2 Stagnation State</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.4 T-s and P-v Diagrams in Compressible Flows</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>23</td>
</tr>
</tbody>
</table>

Exercises ix
### 3 Normal Shock Waves

3.1 Governing Equations .......................... 25
3.2 Mathematical Derivation of the Normal Shock Solution .......................... 27
3.3 Illustration of the Normal Shock Solution on T-s and P-v diagrams .......................... 32
3.4 Further Insights into the Normal Shock Wave Solution .......................... 34
Exercises ........................................ 37

### 4 Flow with Heat Addition- Rayleigh Flow

4.1 Governing Equations .......................... 39
4.2 Illustration on T-s and P-v diagrams .......................... 40
4.3 Thermal Choking and Its Consequences .......................... 48
4.4 Calculation Procedure .......................... 52
Exercises ........................................ 55

### 5 Flow with Friction - Fanno Flow

5.1 Governing Equations .......................... 57
5.2 Illustration on T-s diagram .......................... 58
5.3 Friction Choking and Its Consequences .......................... 62
5.4 Calculation Procedure .......................... 62
Exercises ........................................ 67

### 6 Quasi One Dimensional Flows

6.1 Governing Equations .......................... 69
6.1.1 Impulse Function and Thrust .......................... 70
6.2 Area Velocity Relation .......................... 71
6.3 Geometric Choking .......................... 73
6.4 Area Mach number Relation for Choked Flow .......................... 75
6.5 Mass Flow Rate for Choked Flow .......................... 76
6.6 Flow Through A Convergent Nozzle .......................... 77
6.7 Flow Through A Convergent Divergent Nozzle .......................... 82
6.8 Interaction between Nozzle Flow and Fanno, Rayleigh Flows .......................... 92
Exercises ........................................ 102

### 7 Oblique Shock Waves

7.1 Governing Equations .......................... 107
7.2 $\theta-\beta-M$ curve .......................... 109
7.3 Illustration of the Weak Oblique Shock Solution on a T-s diagram .......................... 113
7.4 Detached Shocks .......................... 119
7.5 Reflected Shocks 121
  7.5.1 Reflection from a Wall 121
  Exercises 123

8 Prandtl Meyer Flow 125
  8.1 Propagation of Sound Waves and the Mach Wave 126
  8.2 Prandtl Meyer Flow Around Concave and Convex Corners 129
  8.3 Prandtl Meyer Solution 131
  8.4 Reflection of Oblique Shock From a Constant Pressure Boundary 135
  Exercises 137

9 Flow of Steam through Nozzles 139
  9.1 T-s diagram of liquid water-water vapor mixture 141
  9.2 Isentropic expansion of steam 142
  9.3 Flow of steam through nozzles 145
    9.3.1 Choking in steam nozzles 146
  9.4 Supersaturation and the condensation shock 152
    Exercises 159

A Isentropic table for $\gamma = 1.4$ 163
B Normal shock properties for $\gamma = 1.4$ 173
C Rayleigh flow properties for $\gamma = 1.4$ 181
D Fanno flow properties for $\gamma = 1.4$ 191
E Oblique shock wave angle $\beta$ in degrees for $\gamma = 1.4$ 201
F Mach angle and Prandtl Meyer angle for $\gamma = 1.4$ 207
G Thermodynamic properties of steam, temperature table 211
H Thermodynamic properties of steam, pressure table 215
I Thermodynamic properties of superheated steam 219

Index 227
CHAPTER 1

INTRODUCTION

Compressible flows are encountered in many applications in Aerospace and Mechanical engineering. Some examples are flows in nozzles, compressors, turbines and diffusers. In aerospace engineering, in addition to these examples, compressible flows are seen in external aerodynamics, aircraft and rocket engines. In almost all of these applications, air (or some other gas or mixture of gases) is the working fluid. However, steam can be the working substance in turbomachinery applications. Thus, the range of engineering applications in which compressible flow occurs is quite large and hence a clear understanding of the dynamics of compressible flow is essential for engineers.

1.1 Compressibility of Fluids

All fluids are compressible to some extent or other. The compressibility of a fluid is defined as

$$
\tau = -\frac{1}{v} \frac{\partial v}{\partial P},
$$

(1.1)

where $v$ is the specific volume and $P$ is the pressure. The change in specific
volume corresponding to a given change in pressure, will, of course, depend upon the compression process. That is, for a given change in pressure, the change in specific volume will be different between an isothermal and an adiabatic compression process.

The definition of compressibility actually comes from thermodynamics. Since the specific volume \( v = v(T, P) \), we can write

\[
\begin{align*}
    dv &= \left( \frac{\partial v}{\partial P} \right)_T dP + \left( \frac{\partial v}{\partial T} \right)_P dT.
\end{align*}
\]

From the first term, we can define the isothermal compressibility as

\[
-\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_T
\]

and, from the second term, we can define the coefficient of volume expansion as 

\[
\frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P.
\]

The second term represents the change in specific volume (or equivalently density) due to a change in temperature. For example, when a gas is heated at constant pressure, the density decreases and the specific volume increases. This change can be large, as is the case in most combustion equipment, without necessarily having any implications on the compressibility of the fluid. It thus follows that compressibility effect is important only when the change in specific volume (or equivalently density) is due largely to a change in pressure.

If the above equation is written in terms of the density \( \rho \), we get

\[
\tau = \frac{1}{\rho} \frac{\partial \rho}{\partial P},
\]

(1.2)

The isothermal compressibility of water and air under standard atmospheric conditions are \( 5 \times 10^{-10} \text{ m}^2/\text{N} \) and \( 10^{-5} \text{ m}^2/\text{N} \). Thus, water (in liquid phase) can be treated as an incompressible fluid in all applications. On the contrary, it would seem that, air, with a compressibility that is five orders of magnitude higher, has to be treated as a compressible fluid in all applications. Fortunately, this is not true when flow is involved.

### 1.2 Compressible and Incompressible Flows

It is well known from high school physics that sound (pressure waves) propagates in any medium with a speed which depends on the bulk compressibility. The less compressible the medium, the higher the speed of sound. Thus, speed of sound is a convenient reference speed, when flow is involved. Speed of sound in air under normal atmospheric conditions is 330 m/s. The implications of this when there is flow are as follows. Let us say that we are considering the flow of air around an automobile travelling at 120 kph (about 33 m/s). This speed is 1/10th of the speed of sound. In other words, compared with 120 kph, sound waves travel 10 times faster. Since the speed of sound appears to be high compared with the highest velocity in