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FUZZY NEURAL NETWORK THEORY AND APPLICATION

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Table of Contents

Foreword ................................................................. ( xi )
Preface ................................................................. ( xv )

Chapter I Introduction .................................................. ( 1 )
§1.1 Classification of fuzzy neural networks ....................... ( 1 )
§1.2 Fuzzy neural networks with fuzzy operators ................... ( 2 )
§1.3 Fuzzified neural networks ........................................ ( 4 )
  1.3.1 Learning algorithm for regular FNN’s ...................... ( 4 )
  1.3.2 Universal approximation of regular FNN’s ................ ( 6 )
§1.4 Fuzzy systems and fuzzy inference networks ................... ( 7 )
  1.4.1 Fuzzy systems .............................................. ( 8 )
  1.4.2 Fuzzy inference networks ............................... ( 11 )
§1.5 Fuzzy techniques in image restoration ....................... ( 12 )
  1.5.1 Crisp nonlinear filters ................................... ( 12 )
  1.5.2 Fuzzy filters ............................................... ( 13 )
§1.6 Notations and preliminaries ..................................... ( 14 )
§1.7 Outline of the topics of the chapters ......................... ( 17 )
References ............................................................ ( 21 )

Chapter II Fuzzy Neural Networks for Storing and Classifying ( 25 )
§2.1 Two layer max–min fuzzy associative memory ............... ( 26 )
  2.1.1 FAM with threshold ....................................... ( 30 )
  2.1.2 Simulation example ....................................... ( 35 )
§2.2 Fuzzy δ–learning algorithm .................................... ( 36 )
  2.2.1 FAM’s based on ‘\(\lor - \land\)’ ............................ ( 36 )
  2.2.2 FAM’s based on ‘\(\lor - *\)’ ............................. ( 39 )
§2.3 BP learning algorithm of FAM’s ............................... ( 45 )
  2.3.1 Two analytic functions ................................... ( 45 )
## Chapter IV Regular Fuzzy Neural Networks

4.1 Regular fuzzy neuron and regular FNN .......................... (131)

- 4.1.1 Regular fuzzy neuron ........................................... (133)
- 4.1.2 Regular fuzzy neural network ................................. (134)
- 4.1.3 A counter example of universal approximation .......... (135)
- 4.1.4 An example of universal approximation .................. (138)

4.2 Learning algorithms ................................................ (143)

- 4.2.1 Preliminaries ...................................................... (145)
- 4.2.2 Calculus of $\vee - \wedge$ functions ......................... (147)
- 4.2.3 Error function .................................................... (151)
- 4.2.4 Partial derivatives of error function ....................... (153)
- 4.2.5 Learning algorithm and simulation ....................... (155)

4.3 Conjugate gradient algorithm for fuzzy weights .......... (157)

- 4.3.1 Fuzzy CG algorithm and convergence ................. (158)
- 4.3.2 GA for finding optimal learning constant ............ (163)
- 4.3.3 Simulation examples ....................................... (164)

4.4 Universal approximation to fuzzy valued functions ..... (166)

- 4.4.1 Fuzzy valued Bernstein polynomial ...................... (166)
- 4.4.2 Four-layer regular feedforward FNN .................... (170)
- 4.4.2 An example ................................................. (173)

4.5 Approximation analysis of regular FNN ....................... (177)

- 4.5.1 Closure fuzzy mapping .................................... (177)
- 4.5.2 Learning algorithm ....................................... (183)

4.6 Approximation of regular FNN with integral norm ....... (189)

- 4.6.1 Integrable bounded fuzzy valued functions .......... (189)
- 4.6.2 Universal approximation with integral norm .......... (191)

References ................................................................. (194)

## Chapter V Polygonal Fuzzy Neural Networks

5.1 Uniformity analysis of feedforward networks .......... (199)

- 5.1.1 Uniform approximation of four-layer network ........ (200)
- 5.1.2 Uniformity analysis of three-layer neural network .... (207)
§5.2 Symmetric polygonal fuzzy number .................................. (211)
  5.2.1 Symmetric polygonal fuzzy number space ....................... (212)
  5.2.2 Polygonal linear operator ........................................ (217)
  5.2.3 Extension operations based on polygonal fuzzy numbers (220)
§5.3 Polygonal FNN and learning algorithm ................................. (223)
  5.3.1 Three-layer feedforward polygonal FNN ......................... (223)
  5.3.2 Learning algorithm ............................................... (226)
  5.3.3 A simulation .................................................... (231)
§5.4 Approximation of polygonal FNN ...................................... (233)
  5.4.1 I/O relationship analysis of polygonal FNN ...................... (234)
  5.4.2 Approximation of polygonal FNN .................................. (240)
References ........................................................................ (247)

Chapter VI Approximation Analysis of Fuzzy Systems .............. (251)
§6.1 Piecewise linear function ................................................. (252)
  6.1.1 SPLF and its properties ............................................ (252)
  6.1.2 Approximation of SPLF's .......................................... (254)
§6.2 Approximation of generalized fuzzy systems with integral norm (259)
  6.2.1 Generalized Mamdani fuzzy system .............................. (261)
  6.2.2 Generalized T–S fuzzy system .................................... (268)
§6.3 Hierarchical system of generalized T–S fuzzy system .......... (273)
  6.3.1 Hierarchical fuzzy system ......................................... (274)
  6.3.2 Generalized hierarchical T–S fuzzy system .................. (276)
§6.4 Approximation of hierarchical T–S fuzzy system ................. (282)
  6.4.1 Universal approximation with maximum norm ............... (282)
  6.4.2 Realization procedure of universal approximation .......... (285)
  6.4.3 Universal approximation with integral norm ................. (288)
References ........................................................................ (290)

Chapter VII Stochastic Fuzzy Systems and Approximations (296)
§7.1 Stochastic process and stochastic integral ........................... (296)
  7.1.1 Stochastic measure and stochastic integral .................... (297)
  7.1.2 Canonical representation of Brownian motion ............... (299)
§7.2 Stochastic fuzzy systems ................................................. (301)
# Table of Contents

7.2.1 Stochastic T–S system ........................................ (301)
7.2.2 Stochastic Mamdani fuzzy system .......................... (304)
§7.3 Universal approximation of stochastic process by T–S system (307)
  7.3.1 Uniform approximation ....................................... (307)
  7.3.2 Approximation with mean square sense .................. (309)
§7.4 Universal approximation of stochastic Mamdani fuzzy system (315)
  7.4.1 Approximation of stochastic Mamdani fuzzy system ... (315)
  7.4.2 Example ......................................................... (318)
References ............................................................... (321)

Chapter VIII Application of FNN to Image Restoration .... (326)
§8.1 Generalized fuzzy inference network ......................... (327)
  8.1.1 Generalized defuzzification operator ..................... (327)
  8.1.2 Fuzzy inference network ..................................... (332)
  8.1.3 Universal approximation of generalized FINN ........ (335)
  8.1.4 Simulation of system identification ....................... (338)
§8.2 Representation of two-dimensional image by FINN ........ (343)
  8.2.1 Local FNN representation of image ....................... (343)
  8.2.2 Optimal FNN filter ........................................... (348)
  8.2.3 Experiment results ........................................... (350)
§8.3 Image restoration based on FINN ............................ (353)
  8.3.1 Fuzzy partition ............................................... (354)
  8.3.2 Selection type FNN and its universal approximation .. (357)
  8.3.3 A novel FNN filter .......................................... (361)
  8.3.4 Learning algorithm .......................................... (362)
  8.3.5 Simulation examples ......................................... (363)
References ............................................................... (366)

Indices ................................................................. (371)
§I.1 Symbols .......................................................... (371)
§I.2 Terminologies .................................................... (373)
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Foreword

Authored by Professors P. Liu and H. Li, "Fuzzy Neural Network Theory and Application," or FNNTA for short, is a highly important work. Essentially, FNNTA is a treatise that deals authoritatively and in depth with the basic issues and problems that arise in the conception, design and utilization of fuzzy neural networks. Much of the theory developed in FNNTA goes considerably beyond what can be found in the literature.

Fuzzy neural networks, or more or less equivalently, neurofuzzy systems, as they are frequently called, have a long history. The embryo was a paper on fuzzy neurons by my former student, Ed Lee, which was published in 1975. Thereafter, there was little activity until 1988, when H. Takagi and I. Hayashi obtained a basic patent in Japan, assigned to Matsushita, which described systems in which techniques drawn from fuzzy logic and neural networks were employed in combination to achieve superior performance.

The pioneering work of Takagi and Hayashi opened the door to development of a wide variety of neurofuzzy systems. Today, there is an extensive literature and a broad spectrum of applications, especially in the realm of consumer products.

A question which arises is: Why is there so much interest and activity in the realm of neurofuzzy systems? What is it that neurofuzzy systems can do that cannot be done equally well by other types of systems? To understand the underlying issues, it is helpful to view neurofuzzy systems in a broader perspective, namely, in the context of soft computing.

What is soft computing? In science, as in many other realms of human activity, there is a tendency to be nationalistic-to commit oneself to a particular methodology and employ it exclusively. A case in point is the well-known Hammer Principle: When the only tool you have is a hammer, everything looks like a nail. Another version is what I call the Vodka Principle: No matter what your problem is, vodka will solve it.

What is quite obvious is that if A, B, ..., N are complementary methodologies, then much can be gained by forming a coalition of A, B, ..., N. In this perspective, soft computing is a coalition of methodologies which are tolerant of imprecision, uncertainty and partial truth, and which collectively provide a foundation for conception, design and utilization of intelligent systems. The principal numbers of the coalition are: fuzzy logic, neurocomputing, evolutionary computing, probabilistic computing, rough set theory, chaotic computing and machine learning. A basic credo which underlies soft computing is that, in
general, better results can be obtained by employing the constituent methodologies of soft computing in combination rather in a stand-alone mode.

In this broader perspective, neurofuzzy systems may be viewed as the domain of a synergistic combination of neurocomputing and fuzzy logic; inheriting from neurocomputing the concepts and techniques related to learning and approximation, and inheriting from fuzzy logic the concepts and techniques related to granulation, linguistic variable, fuzzy if-then rules and rules of inference and constraint propagation.

An important type of neurofuzzy system which was pioneered by Arabshahi et al starts with a neuro-based algorithm such as the backpropagation algorithm, and improves its performance by employing fuzzy if-then rules for adaptive adjustment of parameters. What should be noted is that the basic idea underlying this approach is applicable to any type of algorithm in which human expertise plays an essential role in choosing parameter-values and controlling their variation as a function of performance. In such applications, fuzzy if-then rules are employed as a language for describing human expertise.

Another important direction which emerged in the early nineties involves viewing a Takaga-Sugeno fuzzy inference system as a multilayer network which is similar to a multilayer neural network. Parameter adjustment in such systems is achieved through the use of gradient techniques which are very similar to those associated with backpropagation. A prominent example is the ANFIS system developed by Roger Jang, a student of mine who conceived ANFIS as a part of his doctoral dissertation at UC Berkeley. The widely used method of radial basis functions falls into the same category.

Still another important direction—a direction initiated by G. Bortolan—involves a fuzzification of a multilayer, feedforward neural network, resulting in a fuzzy neural network, FNN. It is this direction that is the principal concern of the work of Professors Liu and Li.

Much of the material in FNNTA is original with the authors and reflects their extensive experience. The coverage is both broad and deep, extending from the basics of FNN and FAM (fuzzy associate memories) to approximation theory of fuzzy systems, stochastic fuzzy systems and application to image restoration. What is particularly worthy of note is the author’s treatment of universal approximation of fuzzy-valued functions.

A basic issue that has a position of centrality in fuzzy neural network theory—and is treated as such by the authors—is that of approximation and, in particular, universal approximation. Clearly, universal approximation is an issue that is of great theoretical interest. A question which arises is: Does the theory of universal approximation come to grips with problems which arise in the design of fuzzy neural networks in realistic settings? I believe that this issue is in need of further exploration. In particular, my feeling is that the usual assumption about continuity of the function that is approximated, is too weak, and that the problem of approximation of functions which are smooth, rather than continuous, with smoothness defined as a fuzzy characteristic, that
is, a matter of degree, must be addressed.

FNNTA is not intended for a casual reader. It is a deep work which addresses complex issues and aims at definitive answers. It ventures into territories which have not been explored, and lays the groundwork for new and important applications. Professors Liu and Li, and the publisher, deserve our thanks and congratulations for producing a work that is an important contribution not just to the theory of fuzzy neural networks, but, more broadly, to the conception and design of intelligent systems.

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